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ABSTRACT

This guide to accompany "Geometric Excursions" contains all of the student information in SE 015 344 plus additional teacher materials. With each section are listings of objectives, equipment and teaching aids, suggested approaches, and discussion questions. Masters are provided for making transparencies and student copies of patterns for three-dimensional solids. Related documents are SE 015 334 - SE 015 344, SE 015 346, and SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

GEOMETRIC EXCURSIONS

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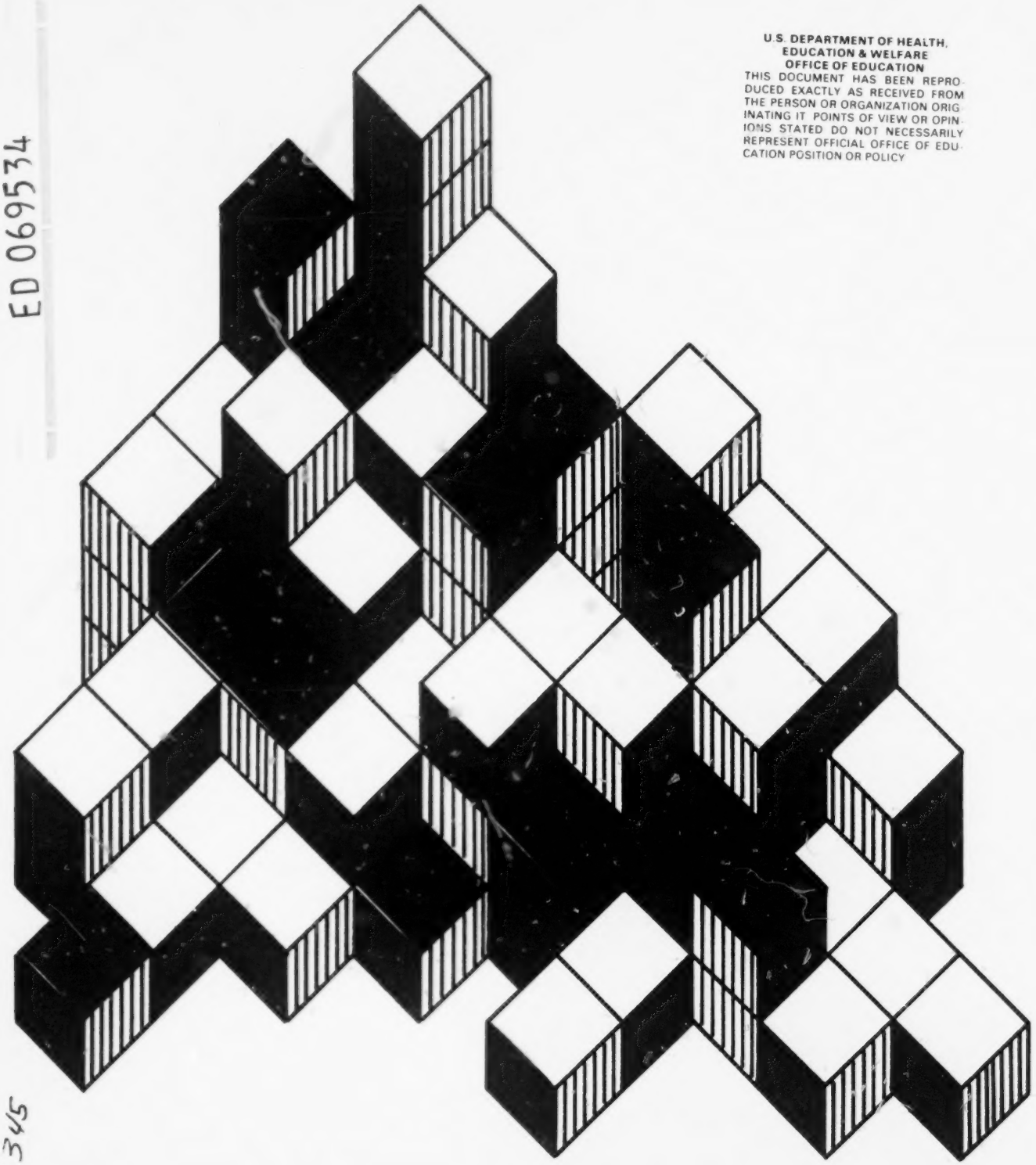

TEACHERS GUIDE

SE 015 345

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GEOMETRIC EXCURSIONS

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

Geometric Excursions begins an approach to the study of non-metric geometry which is different from most treatments of the subject. Instead of starting with the abstract idea of a point and proceeding to a line, to the plane and then to space, the booklet starts with the study of space figures, which it would seem are more natural to the student's experience. The unit provides the students with an opportunity to "do" geometry rather than absorb terminology. You will probably notice the absence of definitions. In most cases, rather than stating a rigorous definition the booklet gives the students a chance to formulate their own definitions through categorical reasoning (these are, these are not, which of the following are). Along with the study of the properties of space figures, Geometric Excursions provides many opportunities to develop space perception.

It is hoped that this unit will: (1) guide the students to perceive the world they live in with a "geometric" eye, and (2) act as a motivational factor for the study of mathematical concepts in the rest of the ninth grade sequence.

BASIC ASSUMPTIONS

It is assumed that: (1) the students have the ability to recognize physical representations of a cube, rectangular prism (box), cone, cylinder, sphere (ball) and pyramid; (2) the student has been exposed to the vocabulary of informal plane geometry.

(Field testing of the original version of this booklet revealed that for most students it is not reasonable to assume number 2. A worksheet which reviews some of the basic terms of plane geometry is included with this revision.)

PREFACE

Fasten your seatbelts and prepare for takeoff! We are ready to begin an excursion into the world of geometry.

The first stop on our journey will be the city of Geopolis where we hope you will pay particular attention to the city's architecture. Before we leave Geopolis you and your classmates will be asked to construct a model of the city so that it may be referred to later on during the excursion.

As our tour takes you from lesson to lesson you will be given the opportunity to explore the properties of various space figures. This will help you identify various three-dimensional geometric shapes, distinguish one from another and discover relationships between them.

At one point we will stop to test some two-dimensional patterns to see if they can be formed into space figures. We hope this experience will help you develop your geometric imagination.

We will visit Sugar Lake City where cubes will present some puzzling problems. From cubes we'll move to the medium of clay and experiment with slicing through solids.

Before the end of the excursion, you will visit the realm of artists, architects and draftsmen where you will develop some techniques for picturing three-dimensional figures on a flat surface.

Our last stop will be the world of two dimensions where you will meet some of the shapes of Shadowland.

We hope you will enjoy this geometric excursion and we hope it will help you look at the world around you with a "geometric" eye.

OBJECTIVES

Geometric Excursions hopes to: 1) interest the students in the study of geometry and mathematics in general; 2) develop their spatial perception; 3) make them aware of the beauty of the geometric forms in the world about them. The unit intends to give the students a chance to discover relationships which are usually ignored in an informal geometry course.

A summary of terminal objectives appears after the TABLE OF CONTENTS. Specific behavioral objectives are listed lesson by lesson. These objectives should not limit the scope of the teaching process.

BOOKLET ORGANIZATION

Material in the Student Booklet is arranged under the headings: Class Activity, Think About It..., Exercises, On your Own, ✓ Point and Taking a Number Break. Think About It questions are intended for in-class discussion.

The Class Activity sections are problem situations which involve group work and use of manipulative aids. These activities are not appropriate for homework assignments for they benefit from inter-group communication. The teacher acts as a resource person and director of learning. Students should be allowed to carry out the activity and make discoveries on their own.

Those sections labeled Exercises are to be used for class work with students sharing ideas and reaching conclusions. These are different from the On Your Own sections where the students should work individually to make discoveries and reach conclusions.

The ✓ Points should be used by the students to check their own progress. They are not intended to be quizzes given by the teacher.

The Taking a Number Break sections are included in order to review some basic computational skills with whole numbers.

The material in the Teacher's Guide is arranged, for each lesson, under the headings: Objectives, Equipment and Teaching Aids, Content and Approach, and Things to Discuss. Most of the answers are printed in blue on the student page and others can be found in the margins around the reduced student pages.

Comments under Content and Approach are intended to portray the flavor of the lesson and to indicate the emphasis and direction. Questions and statements under Things To Discuss may be used to review or extend the lesson.

The Objectives and the Equipment and Teaching Aids for each lesson can be found on the yellow pages at the beginning of each lesson. Masters which can be used to make transparencies or to reproduce copies for the students are printed on the yellow pages at the back of the Teacher's Guide.

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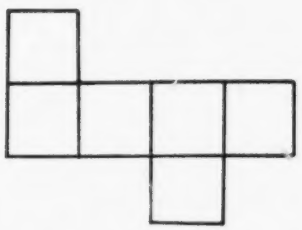
SUMMARY OF TERMINAL OBJECTIVES

1. Given a set of three-dimensional figures, the students will be able to select those which are polyhedra.
2. Given a set of polyhedra, the students will be able to select those which are:
   (a) regular polyhedra
   (b) semi-regular polyhedra
   (c) star polyhedra
   (d) pyramids
   (e) prisms
3. Given a two-dimensional pattern such as the one shown.



the students will be able to tell whether the pattern can be folded into a specified space figure.

4. Given a cube, octahedron or tetrahedron with the middle points of each face indicated, the students will be able to select, from a set of space figures, the figure which would be formed by connecting the middle points of adjacent faces.
5. Given a table of data, the students will be able to develop the relationship connecting the faces, vertices and edges of a regular polyhedron ($F + V = E + 2$).
6. Given a picture of a space figure and a plane passing through it, the students will be able to identify the shape of the section cut by the plane.
7. The students will be able to draw recognizable pictures of cubes, tetrahedrons, cones, cylinders and rectangular prisms.
8. The student will be able to draw a picture of a cube using the one-point perspective technique and the two-point perspective technique.
9. Given a set of two-dimensional figures the student will be able to select the shapes that would be produced by shining a light on a straw model of a cube.

EQUIPMENT AND TEACHING AIDS

A. STUDENT	LESSON
*1. Straws, pipe cleaners, job cards and blueprints (packets A – H).	1
2. Scissors.	1, 3, 4, 7
3. Straw models which the students build in Lesson 1 (used for reference).	2, 5, 10
*4. Triangular, square and pentagonal panels.	3, 4
*5. Rubber bands.	3, 4
6. Models of the cube and octahedron which the students build in Lesson 3.	4
*7. Yarn and needles.	4
8. Models of figures in exercises 8 – 13 on pages 40 – 42.	5
*9. Sugar cubes (30 cubes per student).	6
*10. Elmer's glue.	6
*11. Clay.	7
*12. Cone and cube patterns.	7
13. Cardboard cylinders.	7
*14. Wires for slicing through clay.	7
15. Masking tape.	7
*16. Engineer's drawing paper (grid on one side, blank on the other).	8, 9
17. Straightedge.	8, 9

* materials provided with the booklet

B. TEACHER	LESSON
1. Overhead projector and screen.	2, 3, 4, 5, 6, 9, 10
2. Prepared transparencies $G_1 - G_{17}$.	2, 3, 4, 5, 6, 9
*3. Extra straws and pipe cleaners which can be used to build models of space figures other than the models built by the students in lesson 1.	1, 2, 5
*4. Extra sugar cubes (needed for the Puzzle Enthusiast section, page 49.)	6
*5. Masters ($M_1 - M_{10}$)- See back of Teacher's Guide.	2, 3, 4

NOTE: The TEACHER'S GUIDE for each lesson lists the equipment and teaching aids needed for that lesson.

*materials provided with the booklet

CONTENT AND APPROACH

The students are introduced to the study of non-metric geometry through a look at an imaginary city of 2000 A.D. The buildings of Geopolis were designed to portray some of the space figures which will be studied throughout the booklet.

Let the students read the story and study the layout on pages 2 and 3.

GEOPOLIS: A SPACE CITY OF 2001.

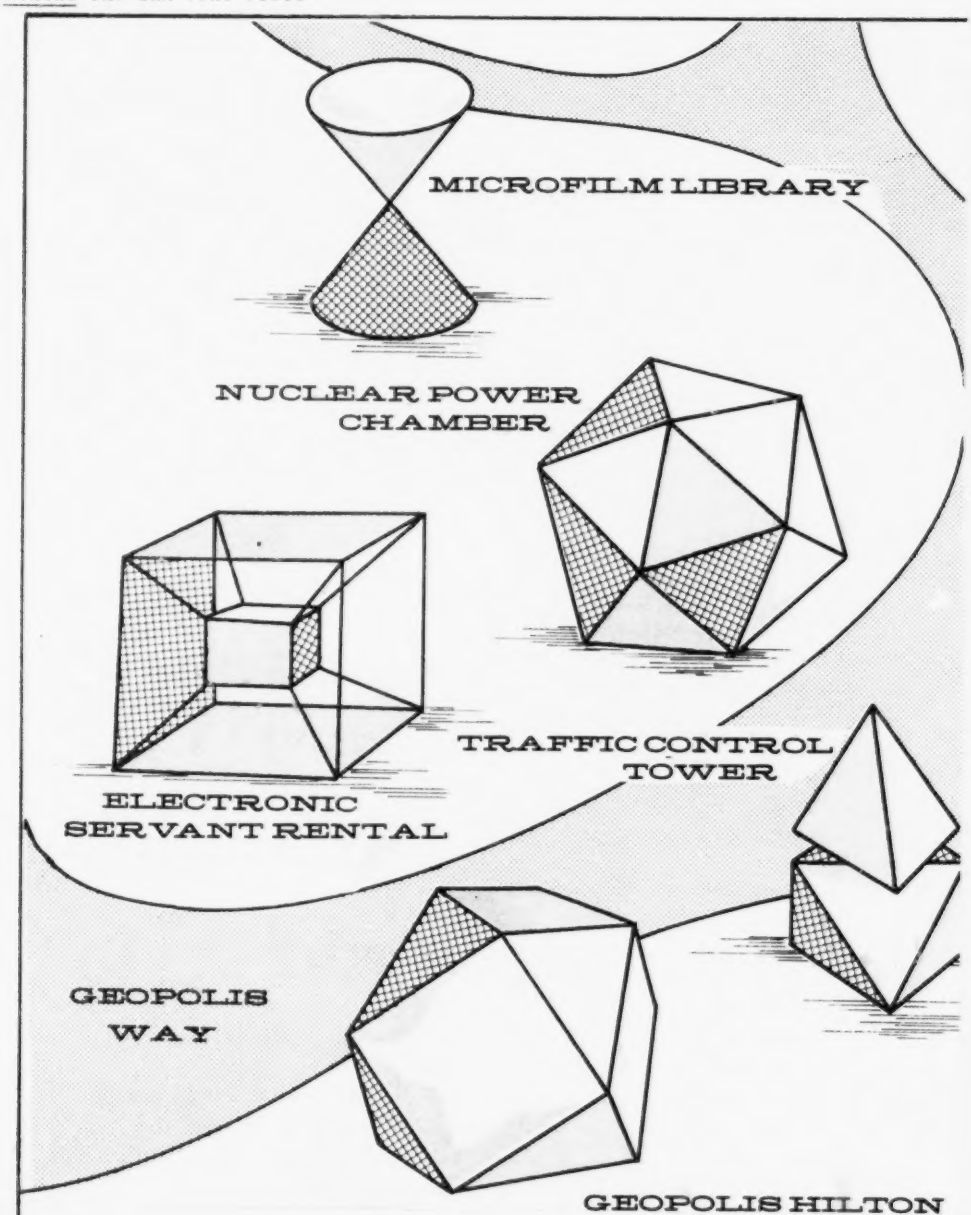
What will life be like in 2001 A.D.? Scientists and futurists predict that it will certainly be different from the world we now live in. They see man living a comfortable life while robots act as servants. They even predict that man will take a pill to improve his "brain-power". Wouldn't that be great — intelligence pills?

People will travel to and from work on monorails or they will drive their cars onto electric highways which resemble huge conveyor belts and glide to work. Hopefully, this will eliminate the agonizing traffic jams we now face on our so-called "expressways". For those people who dislike city-life, they will have an opportunity to get away from it all. They can vacation in space or under the sea. They can even live and work in under-water dwellings which will resemble huge plastic bubbles.

Those people who prefer not to give up star-gazing for bubble-watching will live in huge cities. The city centers will include offices, schools, research centers and control towers. Houses and apartments will be built outside the city center areas. Looking ahead to the year 2001, we tried to visualize one of these city center areas. We created one which we call Geopolis. Study the layout on pages 2 and 3. Then answer the questions on top of page 4.

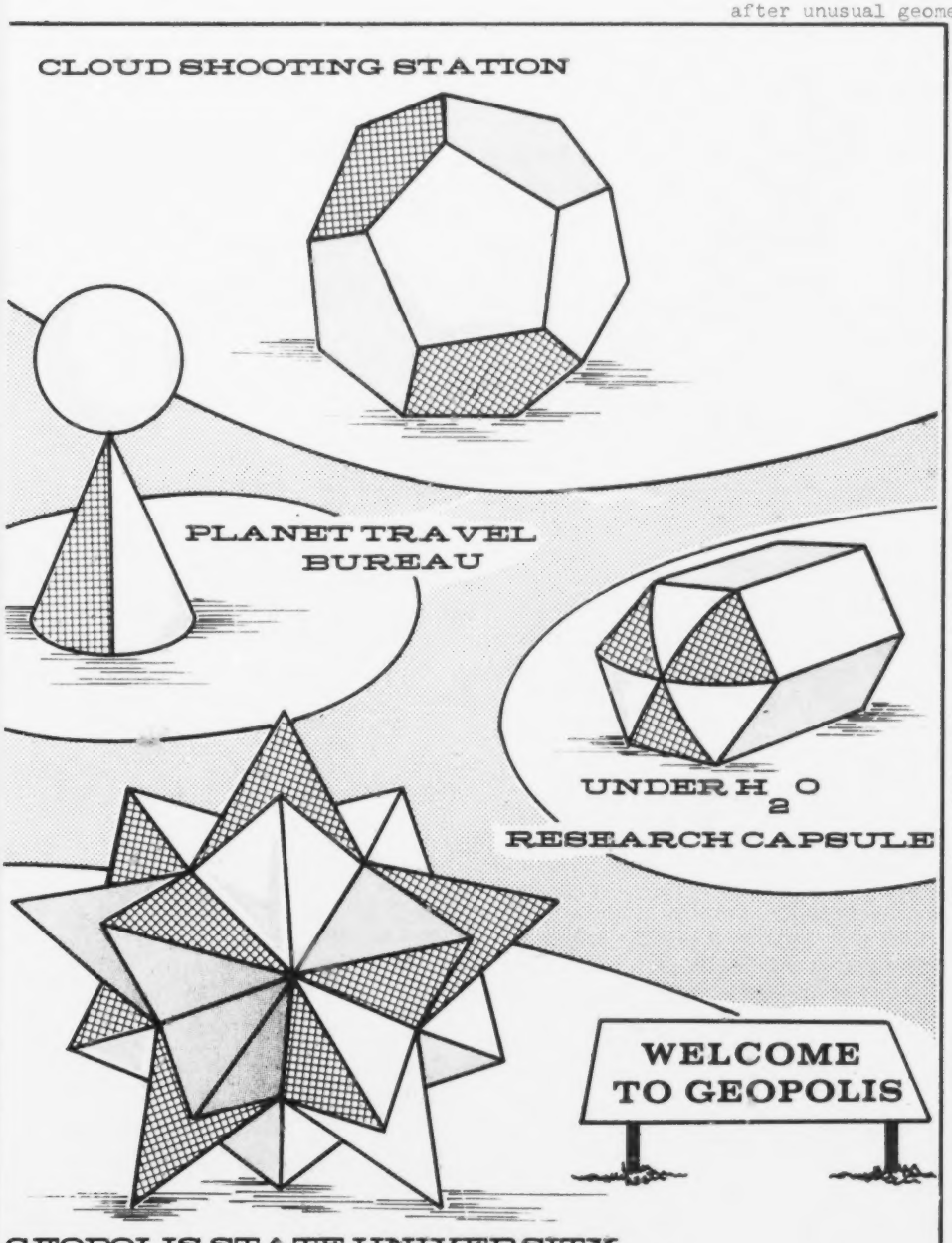
THINGS TO DISCUSS

Your students may wonder about some of the names given to the buildings. The names are designed to go along with some of the predictions that the futurists are making for the year 2000 A.D. The Traffic Control Tower will act much like an airport control tower; it will keep all the conveyor belt roads and the electric cars moving along smoothly. Books, newspapers and documentaries will be printed on minute microfilm reels and stored in the Microfilm Library. At the Electronic Servant Rental one can rent robot servants to carry out the household chores. And, of course, every city must have a Hilton.



THINGS TO DISCUSS

It is said that weather will be controlled by shooting particles into the clouds, thus, Cloud Shooting Station. People will be vacationing on other planets, thus, Planet Travel Bureau. The Under H_2O Research Capsule will be used to carry out experiments to develop foodstuffs from seaweed and other underwater algae.



CONTENT AND APPROACH (For page 4)

Prior to starting the model building activity the class should briefly discuss the Think About It... questions. The questions should remind them that the study of solid geometry is not completely divorced from of the world they live in. For many years architects predominantly used the cube or rectangular prism shape for designing buildings. Modern architectural design is becoming more creative and is getting away form the basic box shape. More and more we can see building patterned after unusual geometric shapes. Could it be that architects are finally taking notice of the trends in modern art?

The student's answers to the questions will vary. There are no set answers. The success of this discussion will depend on the experiences they have had and their observations of the world about them. It would be helpful of you could have in mind a couple of instances where unusual geometric shapes have been used to design buildings in their immediate environment. In answering question 4 they will probably agree that the rectangular (box-shape) is most commonly used. Some possible reasons for this – easy to construct; most efficient use of space; can easily be divided into rectangular-shaped rooms; they are built on rectangular-shaped lots.

CONTENT AND APPROACH

In this Class Activity the students will work in teams to build a facsimile of the city of Geopolis. The straw and pipe cleaner models which they will construct will be useful as they study the concepts in future lessons of this booklet.

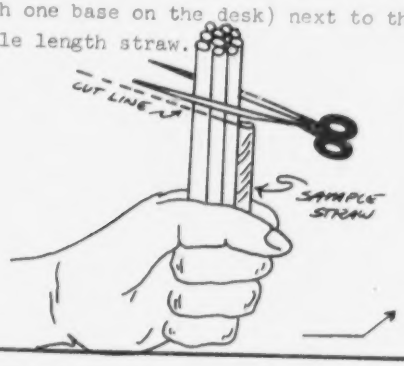
The envelopes (A-H) contain job (instruction) cards, blueprints of the buildings and sample length straw(s) which should be used as models for the edges of the structure. You have received a supply of full length straws which they will have to cut to correspond to the sample lengths. You have also received 12-inch pipe cleaners which will have to be cut into 3-inch pieces. It would facilitate matters if the pipe cleaners could be cut prior to beginning the activity. The most efficient way to do this is with a paper cutter.

The job cards indicate how many straws (sample length) and pipe cleaners (3-inch) are needed to build each particular structure.

The students should figure out how many full length straws they will need to cut to get the desired number of sample length straws.

The following is a suggested technique for cutting the straws. (Ask a student to help you demonstrate it to the entire class.)

1. Hold three or four full length straws. (with one base on the desk) next to the sample length straw.



THINK ABOUT IT ...

1. If you were in charge of designing Geopolis, what additions or deletions would you make?

2. Where (if ever) have you seen the geometric shapes on pages 2 and 3 used in present architecture? Are they commonly used? Why or why not?

3. What other unusual geometric shapes have you seen used in present day architecture?

4. What geometric shapes are most commonly used in designing the buildings of today? Why do you think they are used so often?

CLASS ACTIVITY

The geometric shapes used in designing Geopolis have been purposely selected so that we may study them in future lessons on this booklet. You will work in teams to build a model of Geopolis. Your teacher will pass out the job assignment cards and materials.

CONTENT AND APPROACH

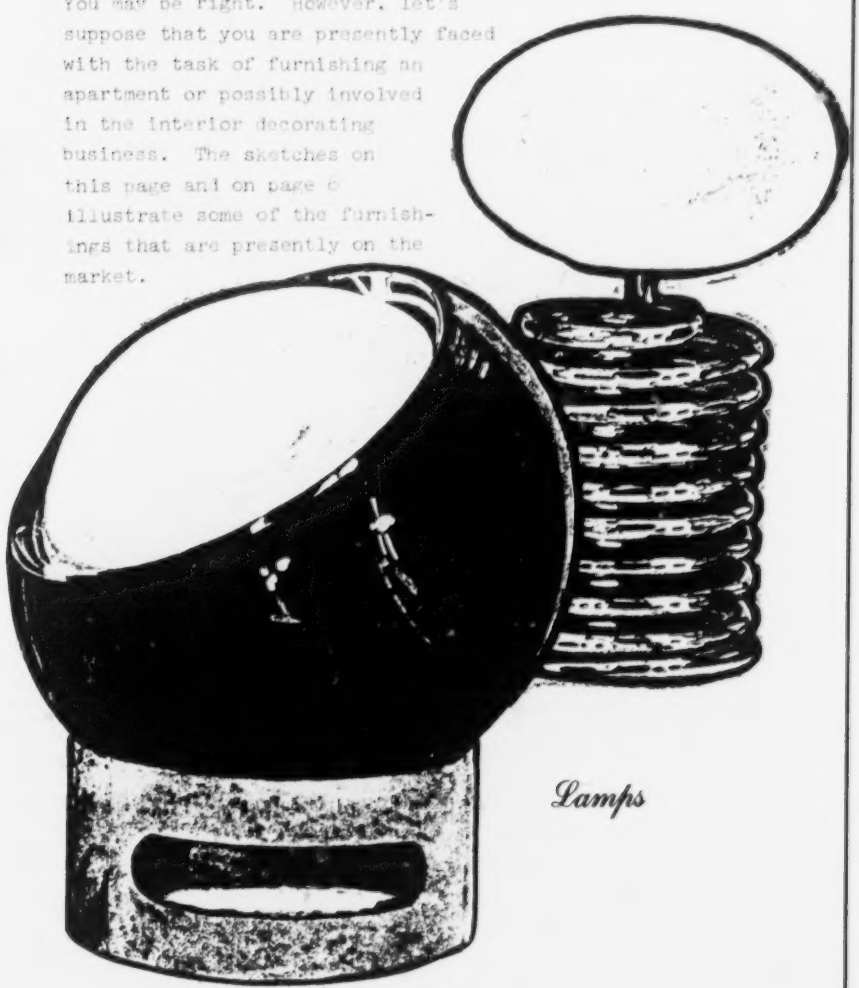
2. One person should hold the straws toward the bottom base and cut along the top of the sample length straw. The other person should hold the tops of the full length straws. This will prevent the straws from flying as the cut is made.

The instructions on the job cards present a production line approach for building the model. Members of the team will have to divide the specific tasks among themselves. Each group should have at lest one pair of scissors. Because of the large number of sample length straws which will be needed for Geopolis State University, team F should have more pairs of scissors at their disposal.

Groups	Suggested Number of Students
A. Traffic Control Tower	2-3
B. Cloud Shooting Station	3-4
C. Nuclear Power Chamber	3-4
D. Electronic Servant Rental	3-5
E. Planet Travel Bureau and Microfilm Library	2
F. Geopolis State University	4-6
G. Under H_2O Research Capsule	2
H. Geopolis Hilton	2-3

SHAPES OF THE 70'S

You probably felt that the architectural design of Geopolis was far-fetched and highly improbable even for 2001 A.D.. You may be right. However, let's suppose that you are presently faced with the task of furnishing an apartment or possibly involved in the interior decorating business. The sketches on this page and on page 6 illustrate some of the furnishings that are presently on the market.



Lamps

Suggestions:

(1) After their models are completed, they can reinforce them by dropping glue in each end of the straws.

(2) For groups who finish quickly, let them experiment with creating other three-dimensional shapes. Possibly you could select some of the figures from lesson 2 (pages 10, 11, 13, 19, 20) or lesson 5 (pages 40, 41, 42) so these models could be utilized by the students when working through the lessons.

For page 5 see comments on page 6.

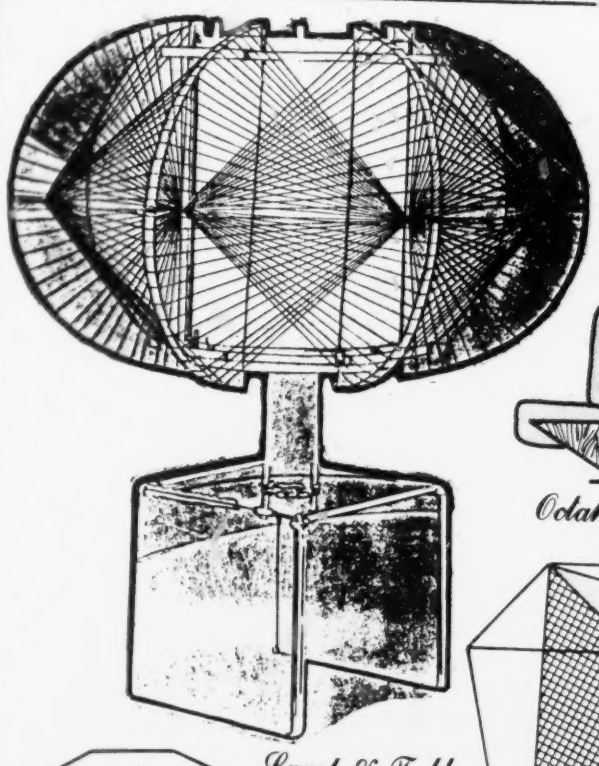
CONTENT AND APPROACH

The shapes used in designing Geopolis were selected purely with the intention of introducing the students to the study of various space figures. At the time of the first printing of this booklet, the author did not realize the relevance of these "way-out" shapes to the interior design of the 70's.

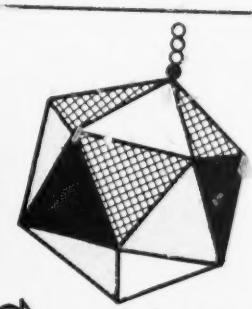
The sketches on this page and on page 5 show furnishings which are now on display in many home furnishing magazines. (Better Homes and Gardens, Ladies Home Journal, etc.)

Most of your students are probably familiar with the lamps at the neighborhood Burger Chef. Ask them if the lamps look like any of the figures on page 2.

You might tell them that Lesson 2 will introduce them to most of these shapes by name so they can impress their friends or even furniture store salesman with comments like: "What a neat icosahedron lamp!", "Do you have any cube-shaped plexiglas tables with a tetrahedron inside?", "Do you carry any octahedron-shaped chairs?".

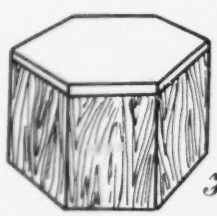


Lamp & Table

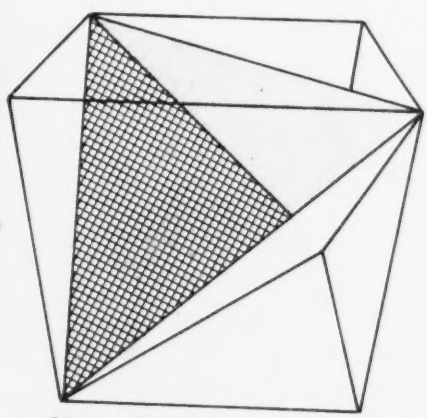


Icosahedron Lamp

Octahedron Chair



Hexagon Table



Cube-Tetrahedron Table

Do these furnishings bear any resemblance to the geometric shapes on pages 2 and 3? If you don't believe the sketches, visit a couple of the furniture stores in your area.

OBJECTIVES

1. Given a set of figures the student should be able to identify the: polyhedra, regular polyhedra, semi-regular polyhedra, star polyhedra, pyramids, prisms.
2. Given a set of properties, the student should be able to select those properties which are characteristic of polyhedra, regular polyhedra, semi-regular polyhedra, pyramids or prisms.
3. Some of the Archimedean (semi-regular) polyhedra are formed by cutting off the corners of a Platonic (regular) polyhedron. Given one of these semi-regular polyhedra the student will be able to identify the regular polyhedron from which it was formed.
4. Five of the star polyhedra are formed by a building pyramids on each face of a regular polyhedron. Given one of these five, the students will be able to identify the regular polyhedron from which it was formed.

EQUIPMENT AND TEACHING AIDS

*1. Straw models from Lesson 1 (for reference)
*2. Clay (Optional)
*3. Prepared transparencies G_1 - G_6
4. Overhead projector and screen
*5. Masters M_1, M_2, M_3, M_6 (Optional-see page 21.)

CONTENT AND APPROACH

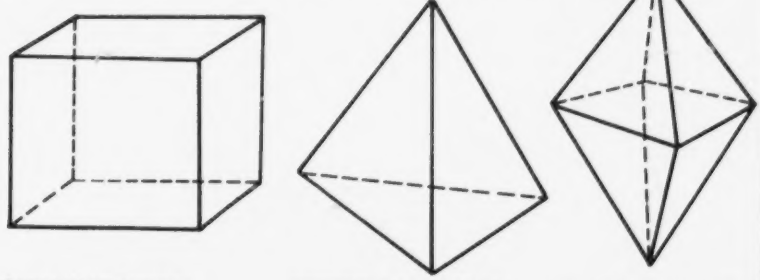
Lesson 2 concentrates on the study of space figures. It begins by introducing the five space figures commonly known in ancient times. (tetrahedron, cube, octahedron, icosahedron and dodecahedron)

If one considers the entire set of three-dimensional geometric figures, the set can be divided into two sets, those which are polyhedra and those which are not. Lesson 2 deals with the set of polyhedra and its special subsets: regular polyhedra, semi-regular polyhedra, star polyhedra, prism and pyramids. The students are given the opportunity to discover the special characteristics of each of these particular subsets through use of categorical reasoning.

The picture of dice at the bottom of the page may not convey to the students the shape of an icosahedron. Use the straw model of the Nuclear Power Chamber (Lesson 1) to display all parts of the figure.

SORTING OUT SOLIDS

Although the shapes used in designing the buildings of Geopolis look very modern and "way-out", they are patterned after shapes which have been known to man for many years. It is said that the ancient Egyptians knew four of them.



HEXAHEDRON TETRAHEDRON OCTAHEDRON

The British Museum has on display a pair of Egyptian icosahedral dice.

ICOSAHEDRONS

THINGS TO DISCUSS

1. What did each of these five space figures symbolize for the Greeks? (tetrahedron-fire; octahedron-air; icosahedron - water; cube-earth; dodecahedron-universe)
2. Can any of these five shapes be found in nature? If so, in what form? (crystals)

CONTENT AND APPROACH

The answer to the question posed at the middle of the page may help the students remember the names of the five regular polyhedra. The names correspond to the number of faces. From Greek:

tetra – four
hexa – six
octa – eight
dodeca – twelve
icosa – twenty

The technique used at the bottom of page 8 (having students identify and discover properties of space figures through the use of categorical reasoning) is introduced here and used throughout the lesson.

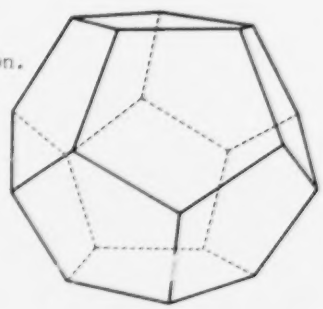
The students should read through pages 8-14 and answer the questions in their own mind. They are asked to jot down their ideas as they read through these pages. Suggest that they circle their answers to the questions: Which of the following are...?

You might let them work through these pages in small groups so that they can help each other identify the characteristics of polyhedra, regular polyhedra and semi-regular polyhedra.

After they have had ample time to do this, they should discuss their findings with the entire class during the class discussion (Think About It) on page 15.

*It would be helpful to the students if you make models of most of the sapce figures pictured on pages 8 - 14 available to them.

The Pythagoreans, a secret society of the Greeks (540 B.C.) discovered the regular dodecahedron.

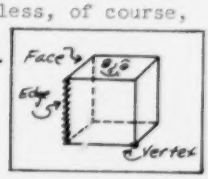


Why do you suppose these shapes were given such strange sounding names? (Use a dictionary to find out.) Which of these five shapes is more commonly known as a cube?

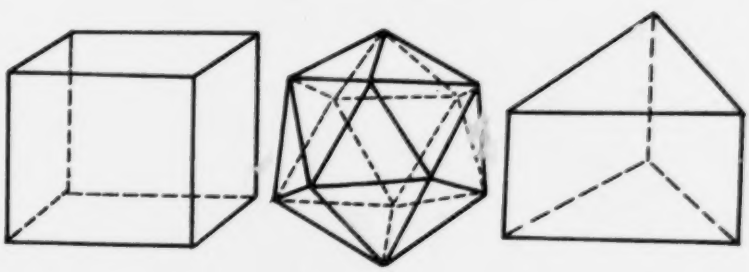
These five figures **(TETRAHEDRON, CUBE, OCTAHEDRON, ICOSAHEDRON, DODECAHEDRON)** belong to a set of three-dimensional geometric figures called Polyhedra.

What are polyhedra? Use the next three sets of figures to answer this question on your own. (Note: the dotted lines show sides which would be hidden from view unless, of course, the figure was made out of straws.)

KEEP THESE TERMS IN MIND



THESE ARE POLYHEDRA.



CONTENT AND APPROACH

These three sets of figures (first set on the bottom of page 6) should convey to the students that a polyhedron is:

(a) a three-dimensional figure whose edges are segments;

(b) a closed surface.

It is important that you point out to your students that there are three legitimate ways of thinking about polyhedra and, more generally, space figures.

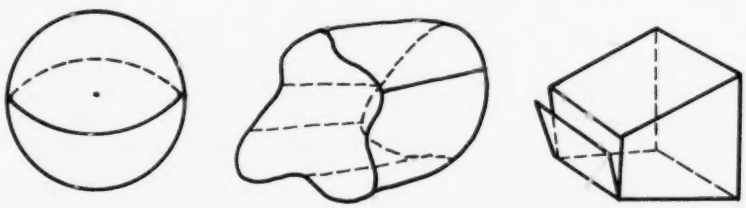
I. The straw (skeletal) models which were built in Lesson 1 can be referred to as polyhedra. In this case the polyhedron is the union of edges (segments).

*II. A polyhedron can also be thought of as a closed surface which is the union of polygonal regions. The models which will be built in Lesson 3 (polygonal panels and rubber bands) will best exemplify this way of describing a polyhedron.

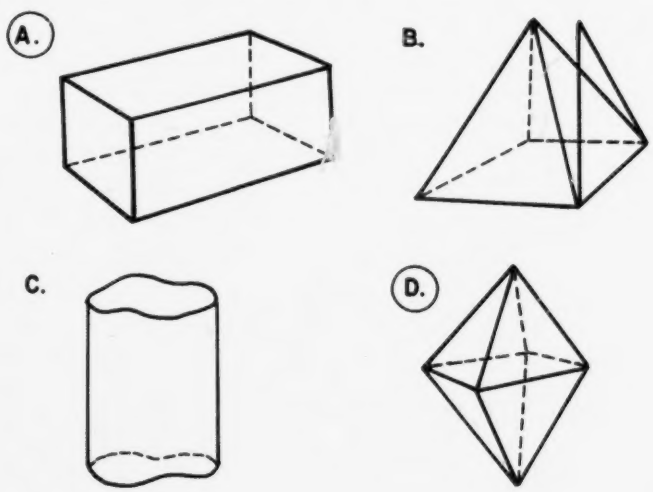
III. Thirdly, a polyhedron can be thought of as the union of the closed surface described in II and all the points on the interior of the closed surface. (e.g., block of wood, sugar cube)

*Geometric Excursions will treat a polyhedron as described in II. This should be made clear to the students. Even though the straw models from Lesson 1 will continue to be used, the students should think of the space figure they represent in terms of a closed surface which is the union of polygonal regions.

THESE ARE NOT POLYHEDRA.



WHICH OF THE FOLLOWING ARE POLYHEDRA?



Jot down some of the properties you would use to identify a polyhedron and go on to the next page.

CONTENT AND APPROACH

These three sets (two on page 10, one on page 11) have been designed to show that a regular polyhedron has:

(1) faces which are the same shape and size;
(2) edges which are the same length;
(3) the same number of edges meeting at each vertex

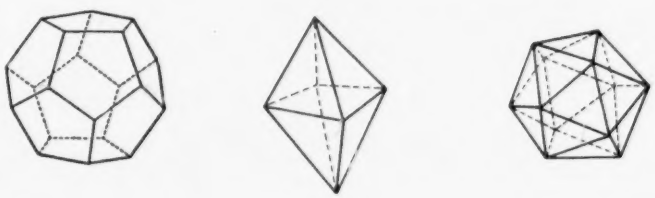
You may want to build straw and pipe cleaner models of some of these to use in the class discussion on page 15. One in particular that you may want to build is pictured in the second set on page 10.

(The faces of this figure are meant to be congruent.)

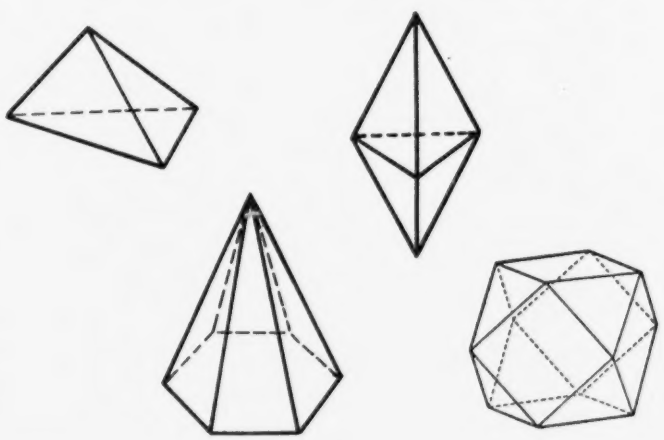
Use straws which are the same length so that the faces are congruent. Present it to the students and go along with the fact that it sure looks like a regular polyhedron (faces congruent). Hopefully, the students will count the number of edges meeting at each vertex of the figure and find out that the number is not always the same.

The set of polyhedra includes a special set of space figures called regular polyhedra. See if you can figure out what the properties of a regular polyhedron are.

THESE ARE REGULAR POLYHEDRA.

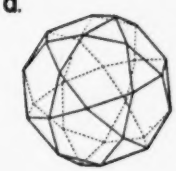


THESE ARE NOT REGULAR POLYHEDRA.

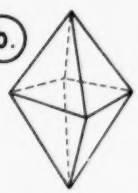


WHICH OF THE FOLLOWING ARE REGULAR POLYHEDRA?

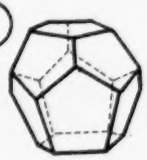
a.



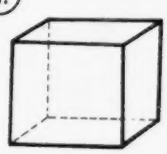
b.



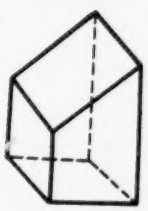
c.



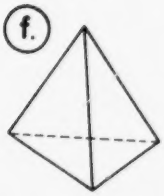
d.



e.

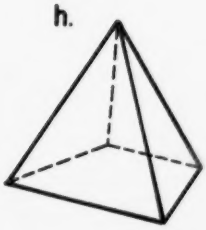


f.

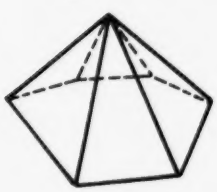


g.

h.



i.



Jot down the ideas you used in making your decision and proceed to page 12.

CONTENT AND APPROACH

Hopefully, the students will realize that the five possible regular polyhedra (Platonic Solids) are the five space figures introduced at the beginning of this lesson — tetrahedron, cube, octahedron, dodecahedron and icosahedron.

THINGS TO DISCUSS

Why are there only five regular polyhedra? (This can be proved using the idea that the sum of the angles at a vertex must be less than 360°. It is not an easy proof but some of your students might like to explore it.)

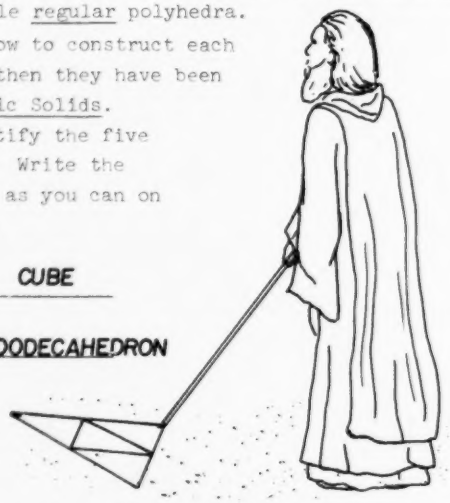
Plato (400 B.C.), a Greek philosopher, said that there are only five possible regular polyhedra. He also showed how to construct each of them. Since then they have been named the Platonic Solids.

Can you identify the five Platonic Solids? Write the names of as many as you can on the lines below.

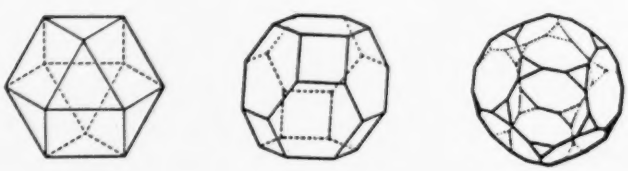
TETRAHEDRON CUBE

OCTAHEDRON DODECAHEDRON

ICOSAHEDRON



Later on, Archimedes (278 B.C.) added thirteen more figures to the set of polyhedra. They are included in the set of semi-regular polyhedra. Three of the thirteen are pictured below:



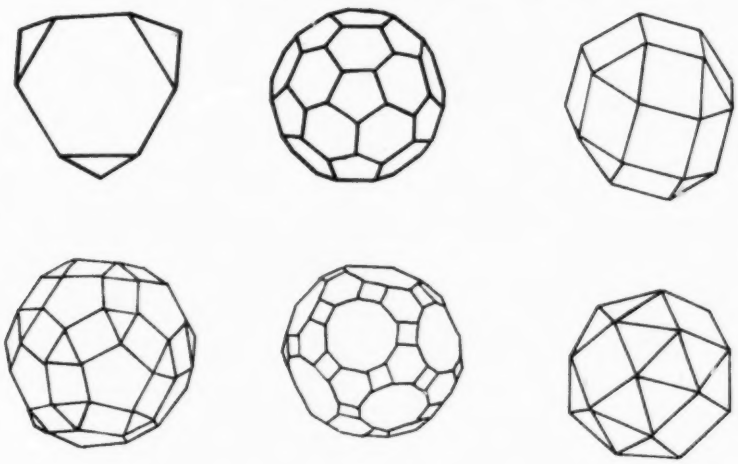
Can you describe a semi-regular polyhedron? If not, the next two pages will help you. If you think you can, record your ideas and then check them by studying the figures on the next two pages.

CONTENT AND APPROACH

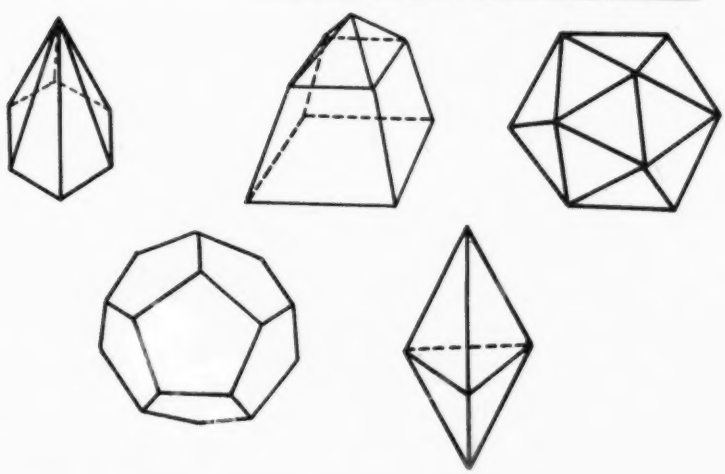
From these two sets of figures the students should discern that the following are characteristics of a semi-regular polyhedron:

a. It has at least two differently shaped faces.
b. The faces which are the same shape are also congruent.
c. The faces are regular polygons.
d. The same number of edges meet at each vertex.

THESE ARE SEMI-REGULAR POLYHEDRA.

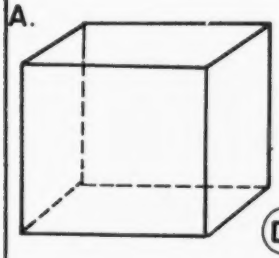


THESE ARE NOT SEMI-REGULAR POLYHEDRA.

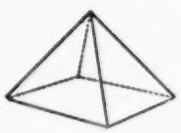


WHICH OF THE FOLLOWING ARE SEMI-REGULAR POLYHEDRA?

A.

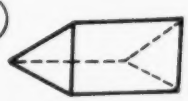


B.

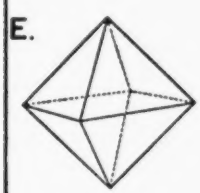


C.

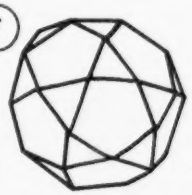
D.



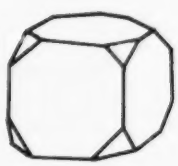
E.



F.



G.



Checklist

Check those properties which you think are characteristic of a semi-regular polyhedra.

✓ 1. The same number of edges meet at each vertex.

✓ 2. Made up of at least two different-shaped faces.

✓ 3. Faces are polygons. **(BUT THEY MUST BE REGULAR POLYGONS)**

✓ 4. Faces are regular polygons.

✓ 5. Like faces are the same size.

____ 6. Like faces are not necessarily the same size.

CONTENT AND APPROACH

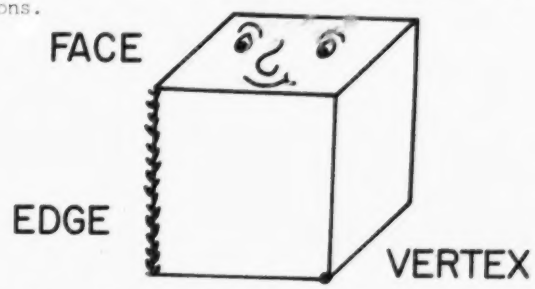
This discussion is most important for it is intended to highlight the properties of the various space figures introduced on the preceding pages.

THINGS TO LOOK FOR IN THE CLASS DISCUSSION (Think About It)

1. faces are polygonal regions; edges are segments (at this point you may want to interject into the discussions the fact that the polyhedra we will be considering are convex.)
2. closed surface
3. least number of vertices - 4; faces - 4; (how about edges?)
4. edges of a regular polyhedron - same length; faces - same size and shape
5. a) faces are not all the same size and shape; at least two different shaped faces - same shaped faces are congruent.
   b) both polyhedra; in some semi-regular polyhedra the edges are the same length; same number of edges meet at each vertex
6. irregular - edges not the same length; faces - not same size and shape.
7. This question is asked just to get them thinking about the fact that there is not a greatest number. If I say 100, you can say 102, etc.

THINK ABOUT IT ...

Use the terms face, edge and vertex in answering the following questions.



1. If a three-dimensional figure is a polyhedron what must be true of its faces? of its edges?
2. What are some of the other characteristics of a polyhedron?
3. What is the least number of vertices a polyhedron can have? the least number of faces?
4. What is true about the edges of a regular polyhedron? about the faces?
5. What makes a semi-regular polyhedron different from a regular polyhedron? How are they the same?
6. What do you suppose are the characteristics of an irregular polyhedron?
7. What is the greatest number of faces a polyhedron can have?

There are many other polyhedra. Some look like stars. Others are called pyramids and prisms. All the polyhedra play an important part in shaping the three-dimensional world we live in.

CONTENT AND APPROACH

(Use G_1 to get the students started on exercise 1.)

This exercise awakens and strengthens the students' spatial perception. They are asked to visualize mentally what the Platonic Solids on page 17 would look like with their corners cut off. Then they match their mental image with the figures on page 16.

The student should think about cutting off the corners of the Platonic Solids. If they do have a difficult time visualizing the "cutting off", you might make a model from clay and actually do the cutting for them.

Besides the spatial perception aspect of this exercise, it exhibits a relationship between the Platonic (regular) polyhedra and a subset of the Archimedean (semi-regular) polyhedra.

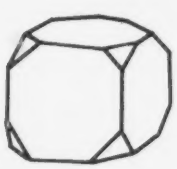
*Hint — the use of clay models shaped like the five Platonic Solids would be helpful for demonstration purposes if your students have trouble with this exercise.

ANSWERS

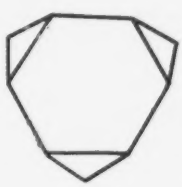
f and g may present a problem. The figure in g is formed by cutting off the corners of the icosahedron or dodecahedron at the mid-points of the edges. The figure in f is formed by cutting off the corners of a cube or an octahedron at the mid-points of the edges.

ON YOUR OWN

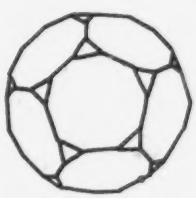
1. Some of the Archimedean (semi-regular) solids are formed by slicing off the corners of the five Platonic Solids. Match each of the semi-regular figures below with the Platonic Solid (on the next page) which was used as its basis. If you're not really sure — guess. (HINT — The numbers corresponding to the figures on page 17 can be used more than once.)



a. 1



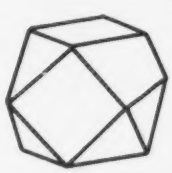
b. 4



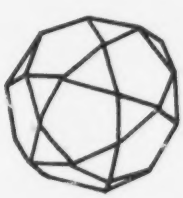
c. 5

d. 2

e. 3



f. 1 or 2



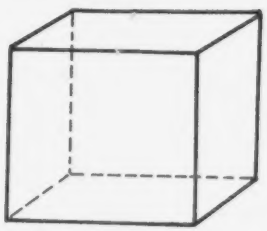
g. 3 or 5

CONTENT AND APPROACH

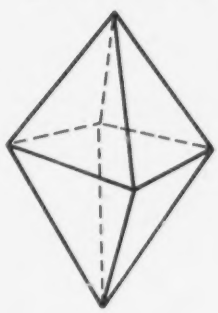
Although it is not a terminal objective, the students will probably use these five basic shapes enough so that they will be able to remember their names.

PLATONIC SOLIDS

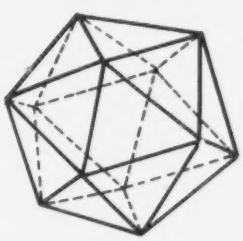
1. CUBE



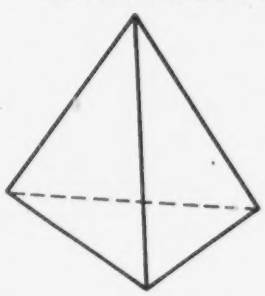
2. OCTAHEDRON



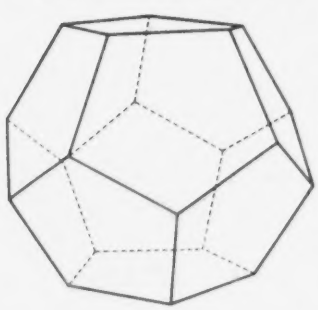
3. ICOSAHEDRON



4. TETRAHEDRON



5. DODECAHEDRON



CONTENT AND APPROACH

Ask your students to formulate their own definition for a pyramid.

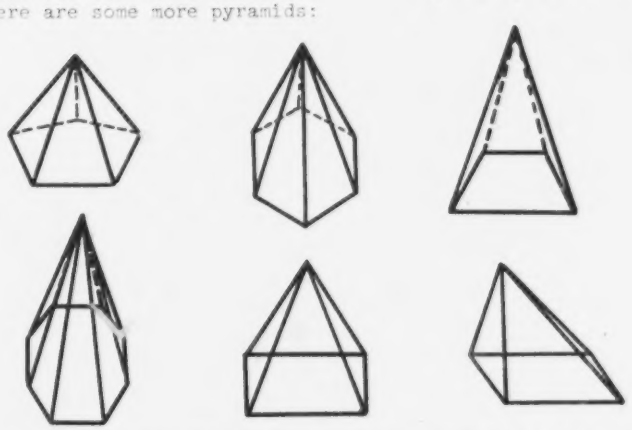
How shall we name the different pyramids? (triangular, square, pentagonal, etc.)

2. The following are pyramids with which you are probably familiar:

Triangular Pyramid

Square Pyramid

Here are some more pyramids:



a. Which of the following are characteristics of a pyramid? (1) (3) (5)

(1) The base is a polygon.
(2) All the edges meet at a point.
(3) All faces other than the base are triangles.
(4) The base is a regular polygon.
(5) All but one vertex lie on the base.

b. (1) What is the smallest number of edges that the base of a pyramid can have? 3

(2) What is the largest number? NO LIMIT

c. How would you distinguish one pyramid from another?

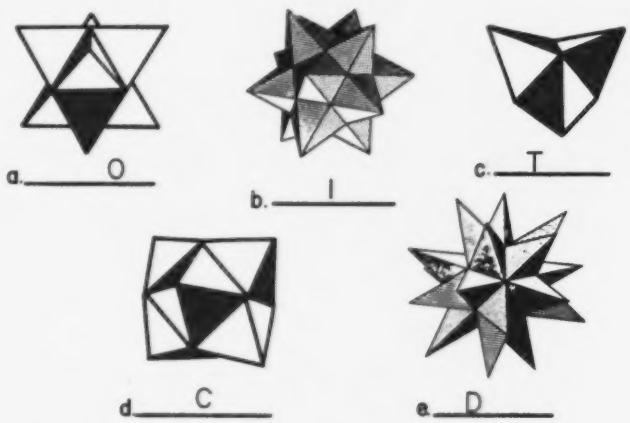
BY THE SHAPE OF THE BASE.

CONTENT AND APPROACH

(Use G_2 to get the students started on exercise 3.)

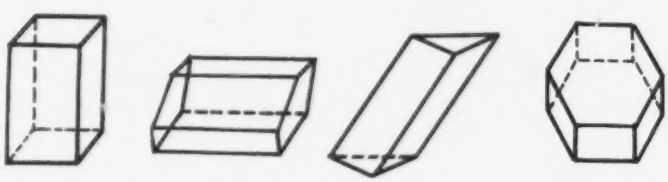
This is another exercise aimed at developing spatial perception. The object is to match each of these star polyhedra with one of the five Platonic Solids pictured on page 17. The leters T, C, O, I, D refer to the names of the Platonic Solids. *Hint — if your students find it difficult to visualize how these star polyhedra can be formed, have them actually build pyramids on the faces of the straw models of the Platonic Solids.

3. The polyhedra pictured below are called star polyhedra. These five were formed by building a pyramid on each face of one of the Platonic Solids (see page 17). Identify the Platonic Solid which was used to form each one. Use the letters T, C, O, I, D to identify the one which you think was used. If you're not sure — Guess! (Hint — Do the number of edges on each face of Plato's polyhedra give you any clues?)



4. Now let's look at a subset of polyhedra which are known as prisms.

   Each of the following are prisms:



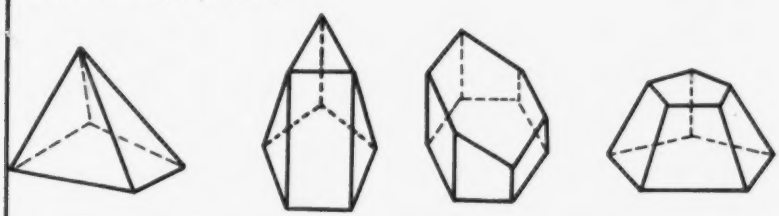
CONTENT AND APPROACH

From the first two sets of figures (the first is on page 19) the students should notice these two characteristics of a prism:

1. At least two faces are congruent and parallel (These are the bases).
2. All faces other than the bases are parallelograms.

Again, it would be helpful to have models of these figures in the classroom.

These are not prisms:



a. Use the preceding sets of figures to decide which of the following statements are true and which are false.

F (1) All the faces of a prism are the same shape and size.

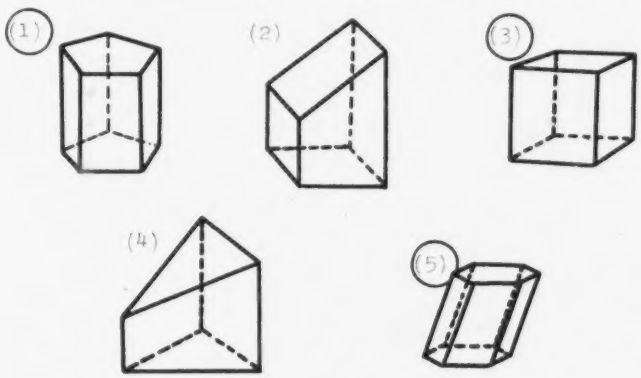
T (2) At least two faces of a prism are parallel and also the same shape and size.

F (3) All but two faces of a prism are rectangles.

F (4) A pyramid can also be a prism.

T (5) All but two faces of a prism are parallelograms.

b. Which of the following are prisms?

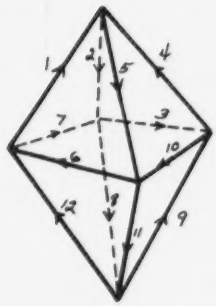


c. Describe a prism in your own words.

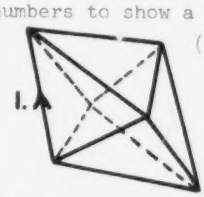
CONTENT AND APPROACH

Exercise 5 introduces the students to network problems. Many of the problems of topology are related to networks. The students are probably familiar with similar problems involving two-dimensional figures. Geometric Excursions II will treat network problems involving plane figures and will give the students more time to discover the relationships between the vertices of closed networks. This exercise is intended to make the students aware of the fact that network problems can also involve space figures.

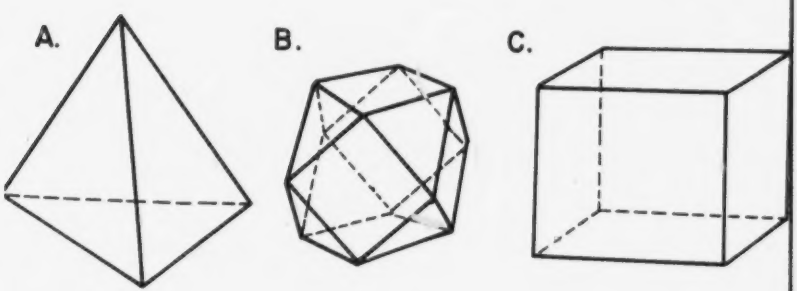
5a.



5a. There are gas fumes escaping from the Traffic Control Tower. The leak is somewhere along one of the edges in the bottom structure. A trouble shooter is sent out to find the crack in the edge. He will move along every edge on the outside of the building. Is it possible for him to move along every edge without going over the same edge twice? If so, show how he can do it. Use arrows and numbers to show a path he might travel. (Caution — use a pencil.)

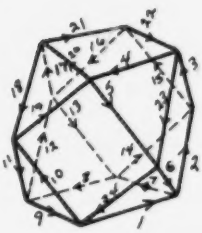


b. Suppose each of the figures below represents a building. For which of the figures is it possible t move along every edge without going over the same edge twice? _____ (If you think the task can be accomplished, show a path which can be taken.)



How can you tell by looking at a figure whether it is possible to move along every edge without going over any edge twice?

5b. the task can only be accomplished in b.



HOW CAN YOU TELL?

If the number of edges which meet at each vertex is even, then it is possible to travel along all the edges of the figure by traversing each edge only once. Every time you go into a vertex you need a path out.

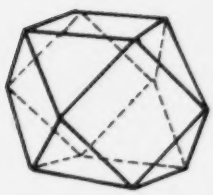
Optional: You might use the masters included at the back of this booklet to run off copies of the figures on this page so your students can use this as a worksheet.

CONTENT AND APPROACH

Question 2 will probably present some difficulties for the students. The best answer is d. Some of your students may select e for it is true that in a regular polyhedron:

1. the same number of edges meet at each vertex.
2. the edges have the same length.

However, a cuboctahedron also satisfies these conditions and it is not a regular polyhedron. It is semi-regular.



Comments to false answers in question 3.

a. There are five.
b. Not always - see pictures at the top of page 13.
c. A pyramid, not a triangle, is built on each face.
e. All but two faces are parallelograms, not necessarily rectangles.
f. A tetrahedron (triangular pyramid) only has three edges on the base.

✓ Point

1. A polyhedron is: a
(select the best answer)

   a. a simple, closed space figure made up of polygons.
   b. any space figure.
   c. any space figure made up of polygons.

2. A regular polyhedron has: d
(select the best answer)

   a. faces which are the same size and shape.
   b. the same number of edges meeting at each vertex.
   c. edges which are the same length.
   d. both a and b. e. both b and c.

3. Mark the following true (T) or false (F):

F a. There are exactly four regular polyhedra.
F b. The edges of a semi-regular polyhedron are all the same length.
F c. One of the star polyhedra can be formed by building a triangle on each face of a cube.
T d. The same number of edges meet at each vertex of a semi-regular polyhedron.
F e. All but two faces of a prism are rectangles.
F f. The base of a pyramid must have at least four edges.
T g. All but one vertex of a pyramid lie on the base.

CONTENT AND APPROACH

These computional exercises give the students more than just rote practice with addition and subtraction of whole numbers. The thought process involved in trying to figure out what number the given letters represent require they do some thinking about the basic ideas of place value. For example:

$$\begin{array}{r} SS \\ +\ W \\ \hline WLL \end{array}$$

What two digit number with both digits the same added to a one digit number will yield a three digit number? If the student selects 88, none of the digits 0-9 when added to 88 will yield a three digit number.

Message

YOU ARE THE WINNER
OF AN ALL EXPENSE PAID
WEEKEND IN BEAUTIFUL
DOWNTOWN GEOPOLIS.

TAKING A NUMBER BREAK

In the following problems, some of the digits in the numbers have been replaced by letters. Figure out which of the digits 0-9 each letter represents. Each letter stands for one and only one digit and different letters stand for different digits.

EXAMPLES:

I. $\begin{array}{r} ZC \\ +\ 17 \\ \hline 35 \end{array}$ $Z = 1$, $C = 8$ $\begin{array}{r} 18 \\ +\ 17 \\ \hline 35 \end{array}$

II. $\begin{array}{r} MM \\ -\ 8 \\ \hline 2J \end{array}$ $M = 3$, $J = 5$ $\begin{array}{r} 33 \\ -\ 8 \\ \hline 25 \end{array}$

a. $\begin{array}{r} 53 \\ -\ NE \\ \hline 17 \end{array}$ $N = \mathbf{3}$, $E = \mathbf{6}$

b. $\begin{array}{r} 34 \\ +\ U \\ \hline AI \end{array}$ $U = \mathbf{7}$, $A = \mathbf{4}$

c. $\begin{array}{r} WL \\ -\ W \\ \hline S \end{array}$ $W = \mathbf{1}$, $L = \mathbf{0}$, $S = \mathbf{9}$

d. $\begin{array}{r} SS \\ +\ W \\ \hline WLL \end{array}$ $S = \mathbf{9}$, $W = \mathbf{1}$, $L = \mathbf{0}$

e. $\begin{array}{r} UTT \\ +\ D2 \\ \hline 774 \end{array}$ $U = \mathbf{7}$, $T = \mathbf{2}$, $D = \mathbf{5}$

f. $\begin{array}{r} IW3 \\ -\ 3IW \\ \hline A32 \end{array}$ $I = \mathbf{8}$, $A = \mathbf{4}$, $W = \mathbf{1}$

Figure out the following message by replacing the numerals with letters. Use your answers to the problems above to make the substitutions. (Caution - Ø stands for the letter Ø and 0 stands for zero.)

Y Ø 7 4 R 6 2 H 6 1 8 3 3 6 R
Ø F 4 3 4 0 0 6 X P 6 3 9 6 P 4 8 5
1 6 6 K 6 3 5 8 3 B 6 4 7 2 8 F 7 0
5 Ø 1 3 2 Ø 1 3 G 6 Ø P Ø 0 8 9

CONTENT AND APPROACH

This lesson presents the students with an opportunity to see how a two-dimensional plan or pattern can be employed to build a three-dimensional object. In the first part of the class activity they will build models of the five Platonic Solids using the illustrated two-dimensional patterns. Once they have had this experience they are asked to discover different patterns for constructing a cube and an octahedron. This together with the On Your Own exercise should further develop their ability to perceive relationships in space.

It is essential that they work in groups so that they can help one another manipulate the panels and rubber bands. It would be a good idea to build a couple of the models prior to conducting the activity so that you will be prepared for any problems that might arise.

THINGS TO DISCUSS

The ideas presented in this lesson have some definite practical applications. Many articles in industry are made from flat sheets of paper, metal or other material. The pieces are first cut into an outline or pattern. They are then rolled, folded or stamped into shape. Finally, certain edges are joined by gluing, soldering, riveting, seaming or welding them together.

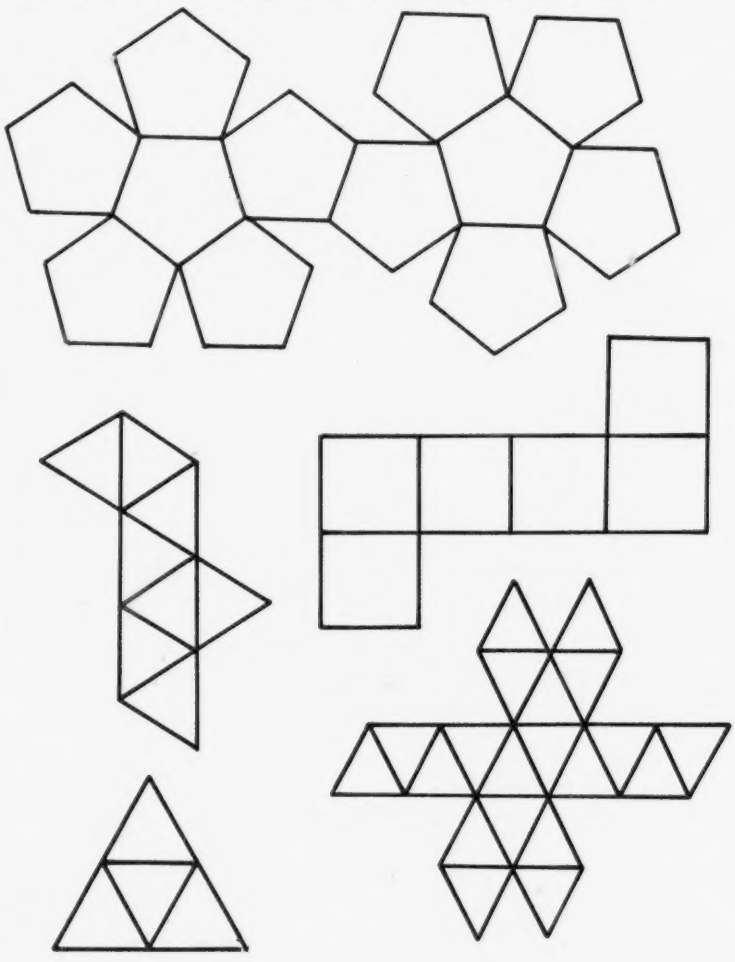
Ask the students to think of some objects which are produced using this technique.

Masters M_7 – M_{10} illustrate patterns used for paper cartons, metal elbow joints and transition pieces which are used to connect pipe or ducts of different sizes. Use the masters to make transparencies.

Drafting teachers or industrial arts teachers would be able to provide other suggestions for supplementing this discussion. They may even want to collaborate with you in presenting this lesson.

POLY PATTERNS

The following are two-dimensional patterns which can be folded to form the three-dimensional Platonic polyhedra:



OBJECTIVES

1. Given a picture of a two-dimensional pattern for one of the Platonic Solids and some cardboard panels and rubber bands, the student will be able to work with two of his classmates to build models of the five solids.
2. The student will be able to work in a group to find patterns (other than the one illustrated) which can be folded to form a cube.
3. The student will be able to work in a group to find patterns (other than the one illustrated) which can be folded to form an octahedron.
4. Given a set of two-dimensional patterns made up of triangles or squares the student will be able (without actually folding) to select those which can be folded to form a tetrahedron, cube or octahedron.

EQUIPMENT AND TEACHING AIDS

*1. Triangular, square and pentagonal panels
*2. Rubber bands
3. Scissors - one for each student
*4. Prepared transparencies G_7 and G_8
5. Overhead projector and screen
*6. Masters M_1 – M_{10} (Optional-see page 34.)

CONTENT AND APPROACH

In part 1 of this Class Activity the students are asked to cut out the panels on their inserts (A, B, C, C). Each student should have two inserts marked C on which are printed pentagonal panels.

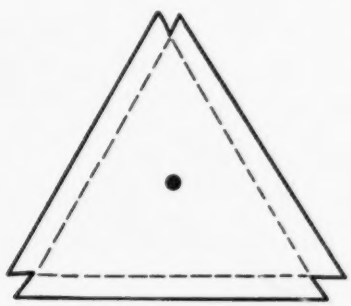
*CAUTION — In order to take the inserts out of their books, ask the students to cut along the dotted lines on the left side of the sheet. The binding will not hold together if they tear the inserts out.

In part 2 they are instructed to pool their panels with two of their classmates. This is necessary because each student does not have a sufficient number of panels to build his own models of the five figures. It is also preferable to work in a group so they can help one another manipulate the panels and rubber bands. You might want to assign groups or let them choose their own groups.

Once they have arranged their panels to look like the patterns on page 24, they can attach the panels with rubber bands and fold their patterns to build models of the Platonic Solids.

CLASS ACTIVITY

1. Cut out the triangular, square and pentagonal panels on the following inserts (A, B, C, C). Fold along the dotted lines to form flaps.



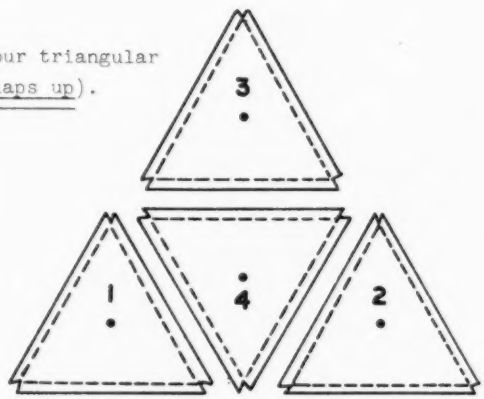
2. Pool your panels with those of two of your classmates. Arrange the panels (flaps up) to form patterns exactly like those on page 24. Before your patterns can be folded to form models of the Platonic Solids, the panels which touch will have to be connected. You will need a supply of rubber bands. Start with the pattern which is like the one in the lower left-hand corner of page 24. One person in the group should hold the pattern in place while the other two put the rubber bands on. After this has been completed, the pattern can be folded to form a tetrahedron. If the above instructions are not clear enough, study the step-by-step instructions which are on the next page.

Step-by-step instructions for building the tetrahedron are printed on page 26.

You might emphasize the point that when they arrange their panels on the desk the flaps should always be up.

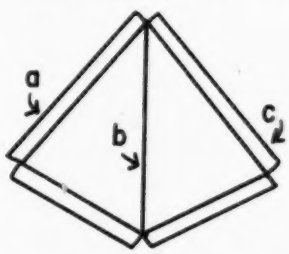
EXAMPLE:

a. Arrange four triangular panels (flaps up).

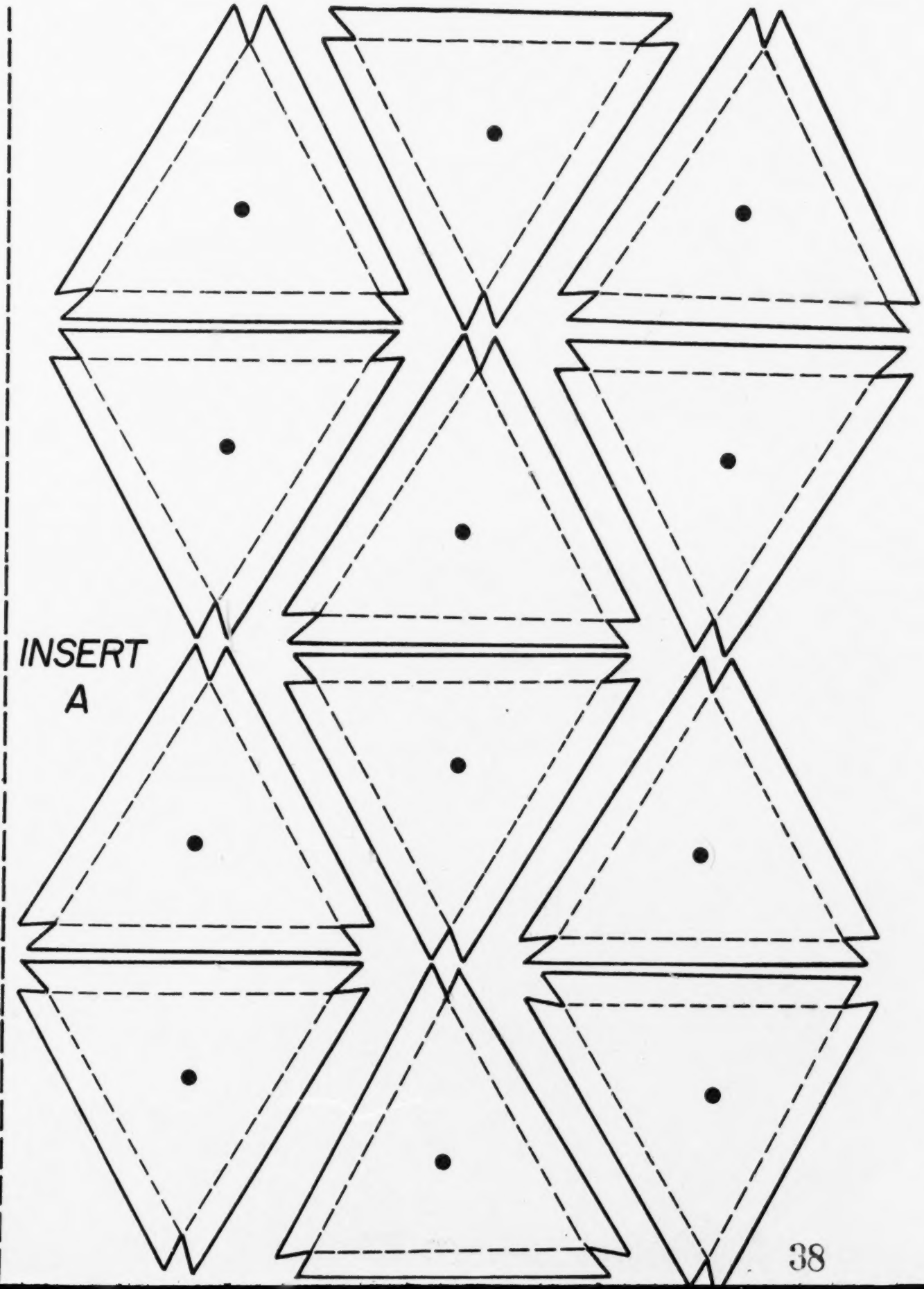


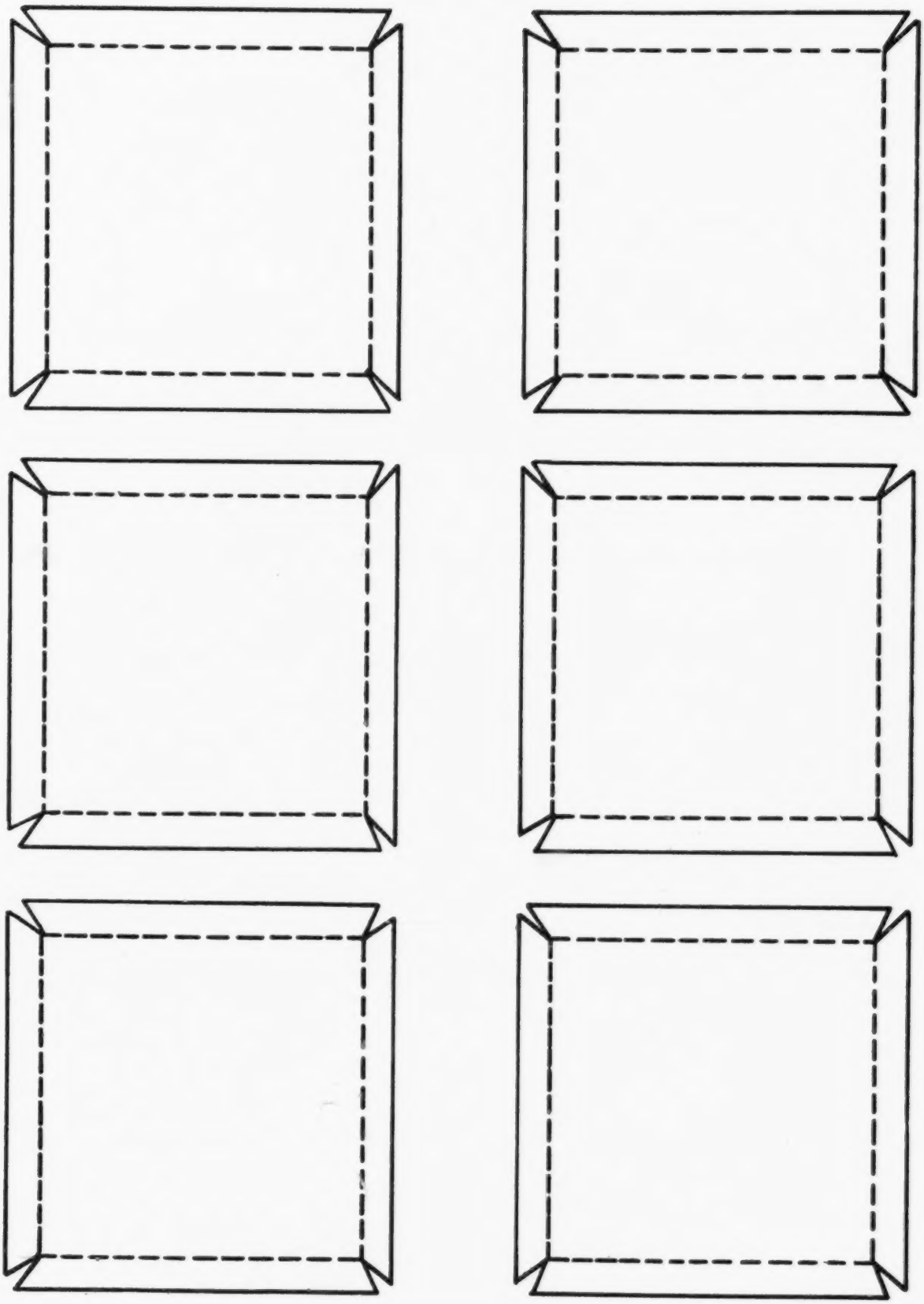
b. Connect the flaps of adjacent panels with a rubber band. (1 to 4; 2 to 4; 3 to 4.)

c. Fold the pattern into the shape of a tetrahedron. Connect the other three edges (a, b, c) with rubber bands.

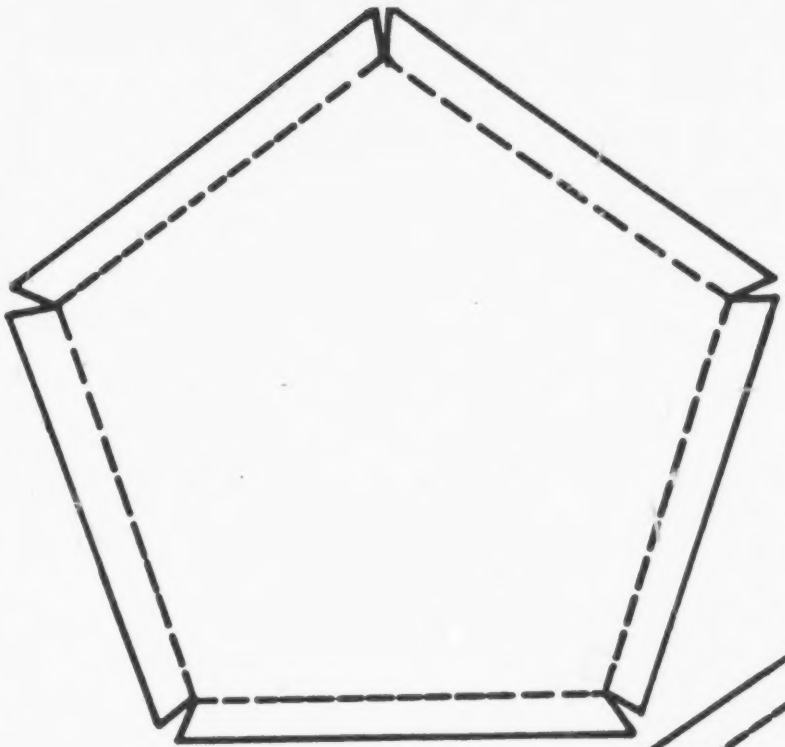


Follow the same procedure with each of the other patterns you have already formed and build models of a cube, octahedron (bottom of Traffic Control Tower), icosahedron (Nuclear Power Chamber) and dodecahedron (Cloud Shooting Station). Don't forget to have one person hold the pattern down while the other two apply the rubber bands.

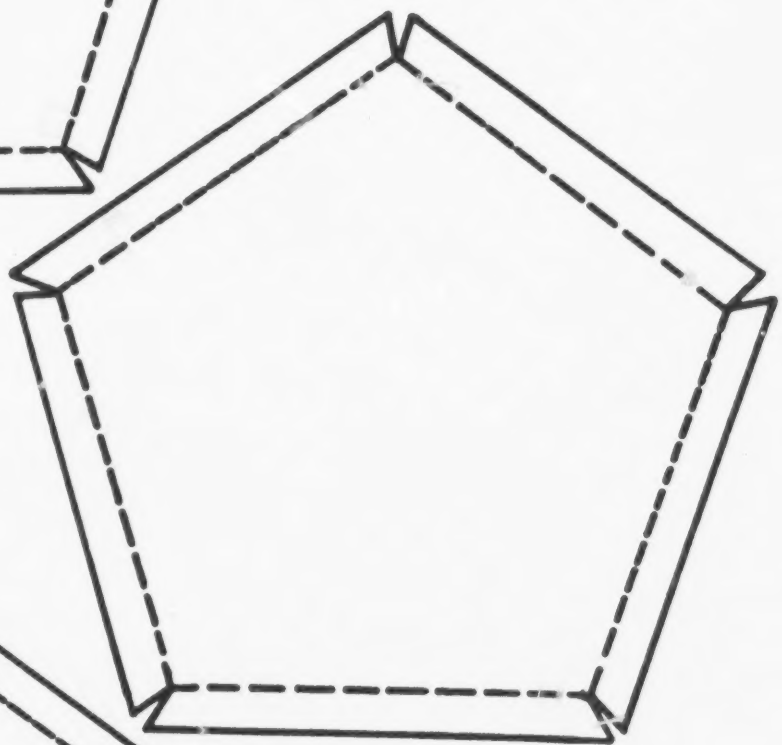

INSERT
A

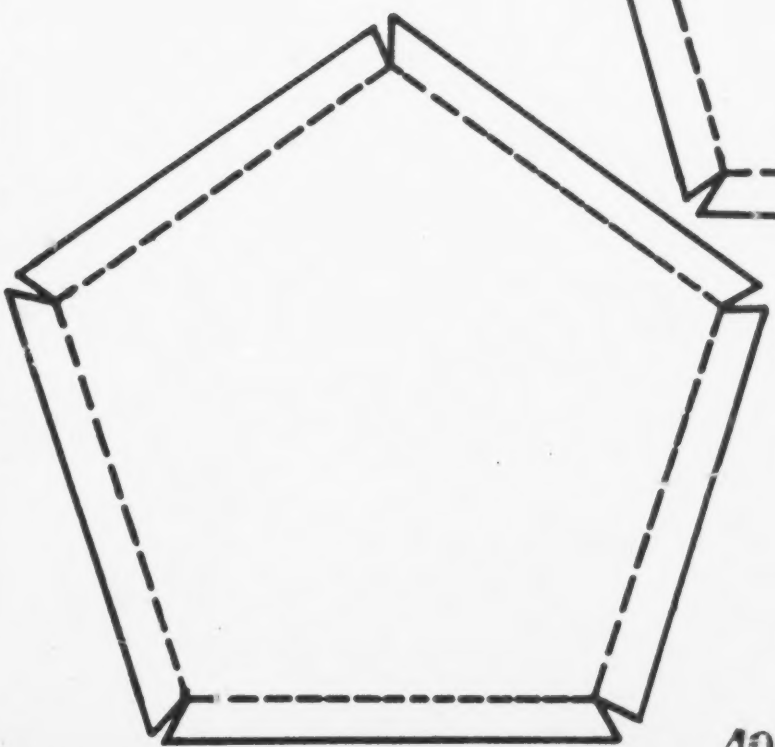


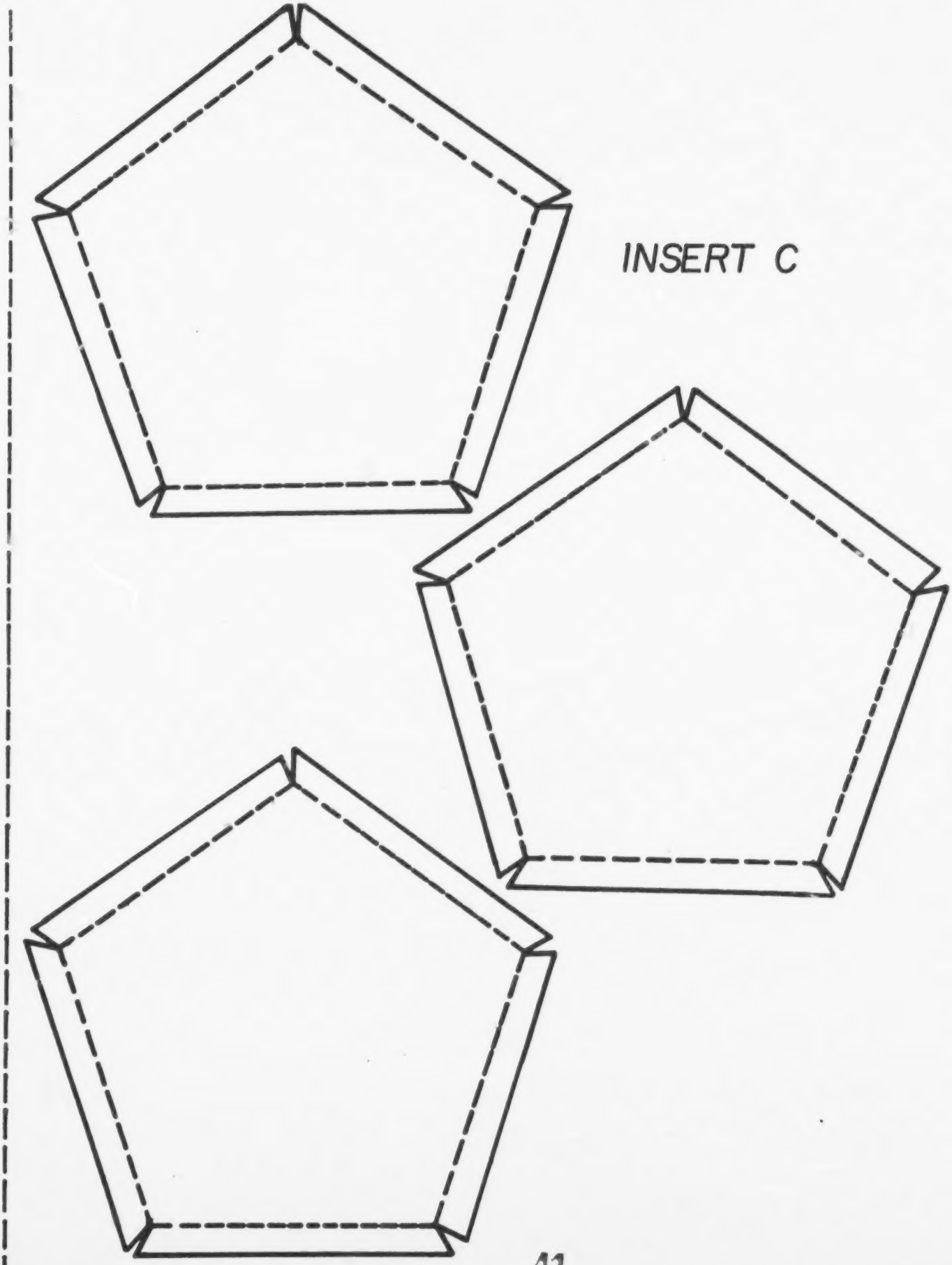
INSERT B



INSERT C






INSERT C

CONTENT AID APPROACH

After the groups have completed their models they should proceed to part 3 where they are asked to find some other two-dimensional patterns which can be folded to form a cube.

The purpose of this activity along with the similar activity in part 4 on page 29 is to further develop their space perception.

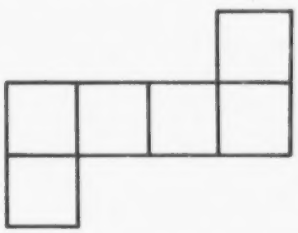
In 3(a) the students are asked to venture a guess at the number of different patterns which they think exist. It would be interesting to canvass each group and record their guesses before they start to work on the problem. Don't reveal the answer.

*Also before they start looking for different patterns point out they should read through the Hints on this page and the Caution on page 28.

The panels and the rubber bands lend themselves to such an activity. Once a pattern is found and folded to form a cube, the students can remove some rubber bands from the model and flatten the connected panels back down into the plane of the desk. If this process does not produce a new pattern they can detach one or more panels and move them around until they come up with a new pattern.

There are <u>eleven</u> different two-dimensional patterns for the cube (discounting all those which can be flipped or rotated to look the same). The patterns have been recorded on your insert AA.

3. To build a model of the cube you started with this pattern:



(a) How many different two-dimensional patterns do you think there are which can be folded to form a cube?

(Guess!) _____

(b) See how many patterns you can find which can be folded to form a cube.

<u>HINTS</u>:

(1.) Take your original model; remove any seven rubber bands (no more than 3 from the edges of the same face). Flatten the panels out on your desk. Is this pattern different?

(2.) Now remove just one panel from the pattern you came up with. Move it around. Connect it in another place. Can this pattern be folded to form a cube?

(3.) Keep moving one or more panels around and record all the different patterns you are sure can be folded to form a cube. (Use insert AA to record your patterns. The original has already been recorded for you.)

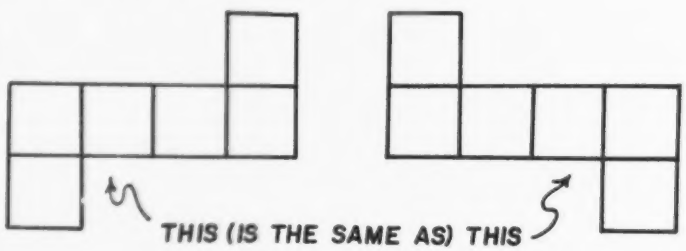
CONTENT AND APPROACH

When the students first start looking for different patterns, they should check their findings by physically folding the pattern to make sure it will form a cube. As they continue their search, however, they should eventually reach a point where they can check by mentally folding the patterns they come up with. As you move around the classroom to survey the progress of the different groups — encourage your students to do their folding mentally. This will help develop their space perception.

After the class has had ample time to carry out this activity, use transparency 3-7 to record their findings. If a group suggests a pattern and someone else challenges it, require that the suggesting group prove that the pattern can be folded to form a cube. Remind them to keep a close watch for patterns which may appear to be different but are actually flips or rotations of one another.

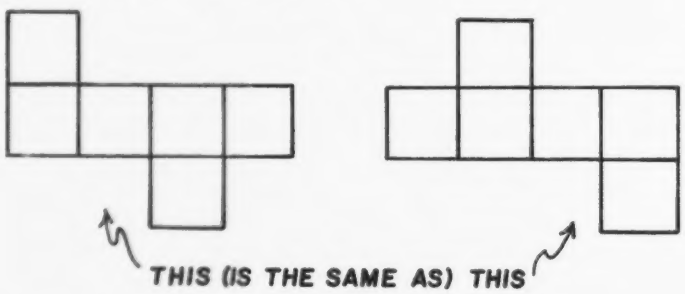
CAUTION:

Make sure each pattern you record is different.



These two patterns are the same. The first can be flipped to get the second.

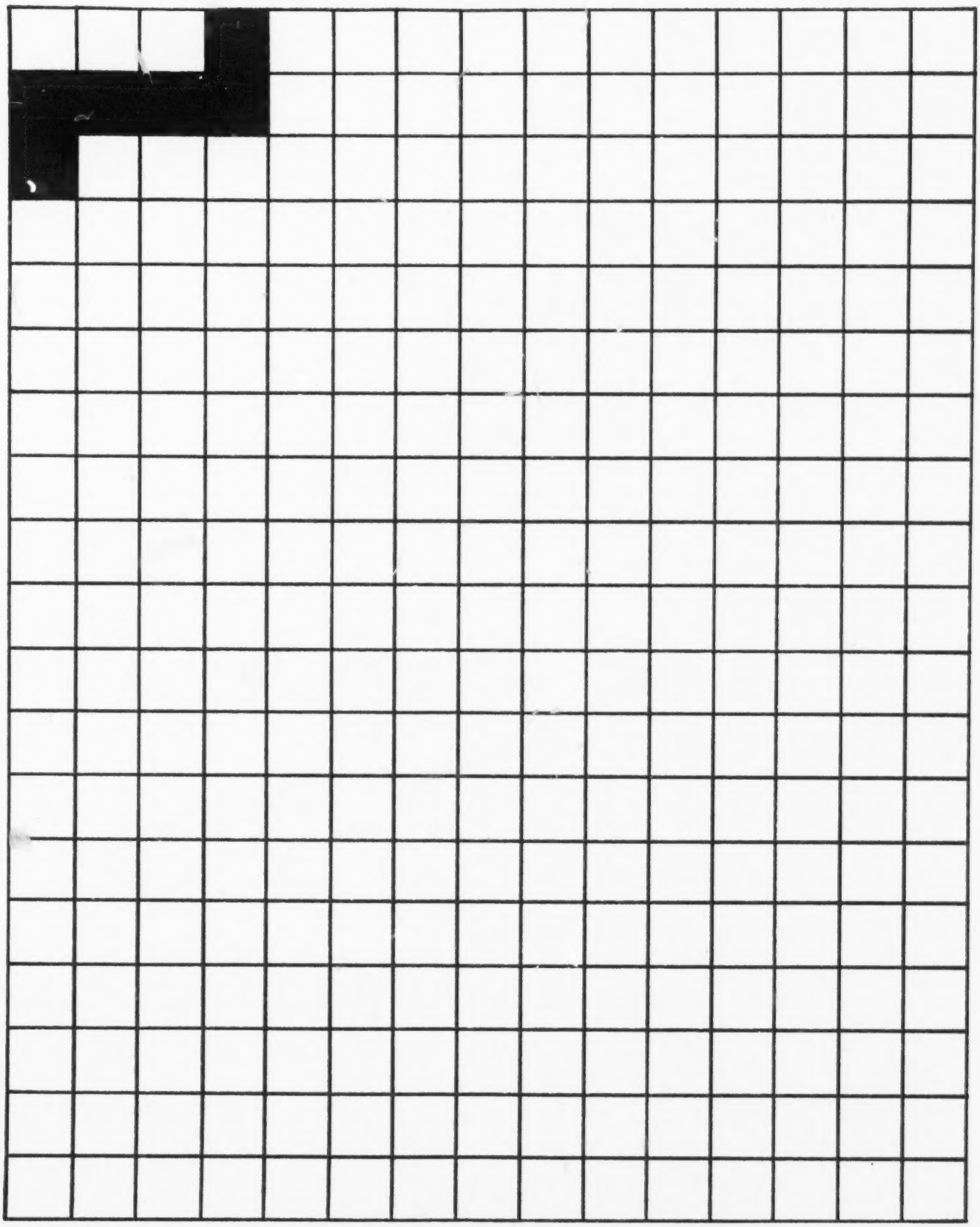
These two are also the same:



The first has been rotated to get the second.

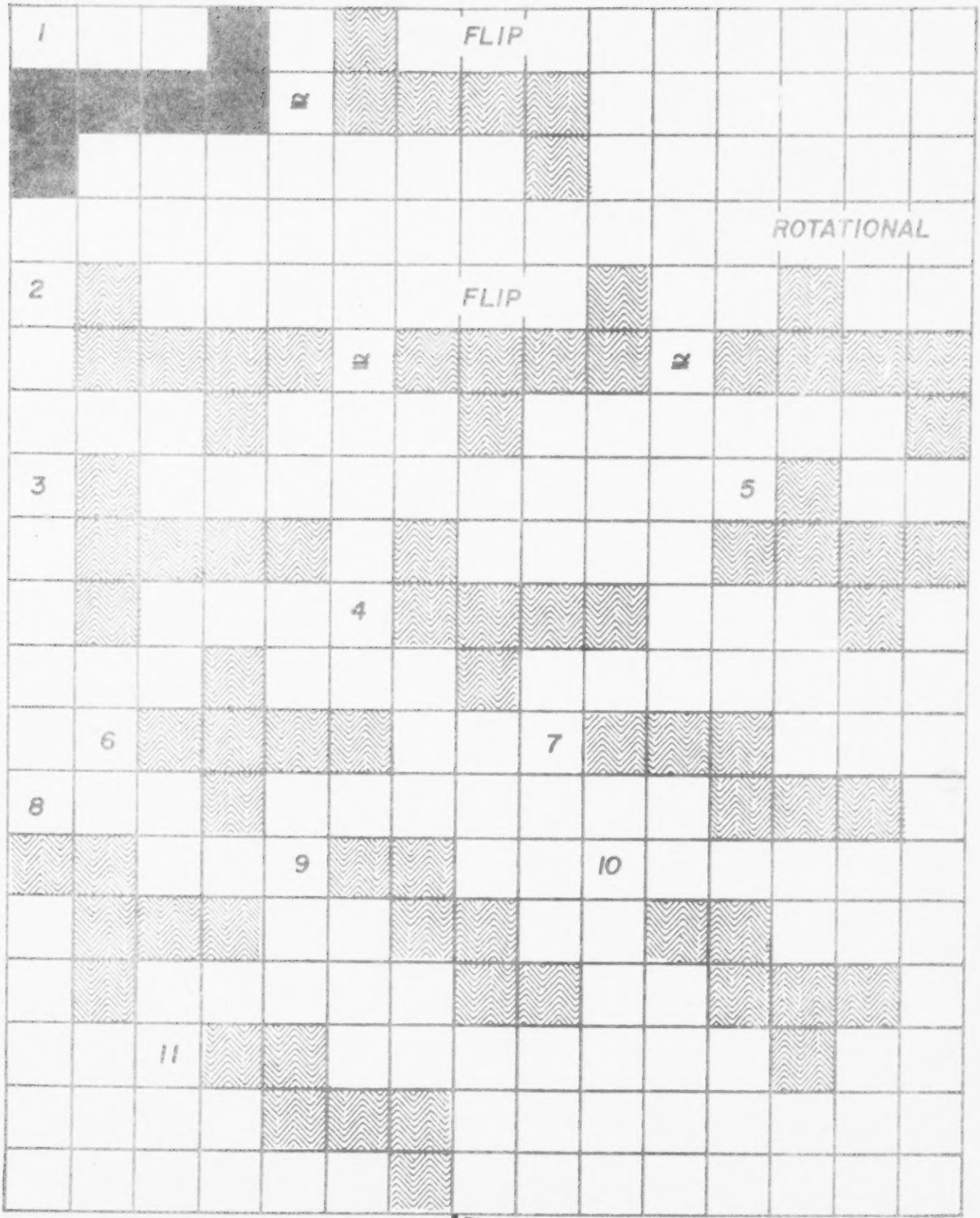
REMEMBER — no flips or rotations.

INSERT A-A

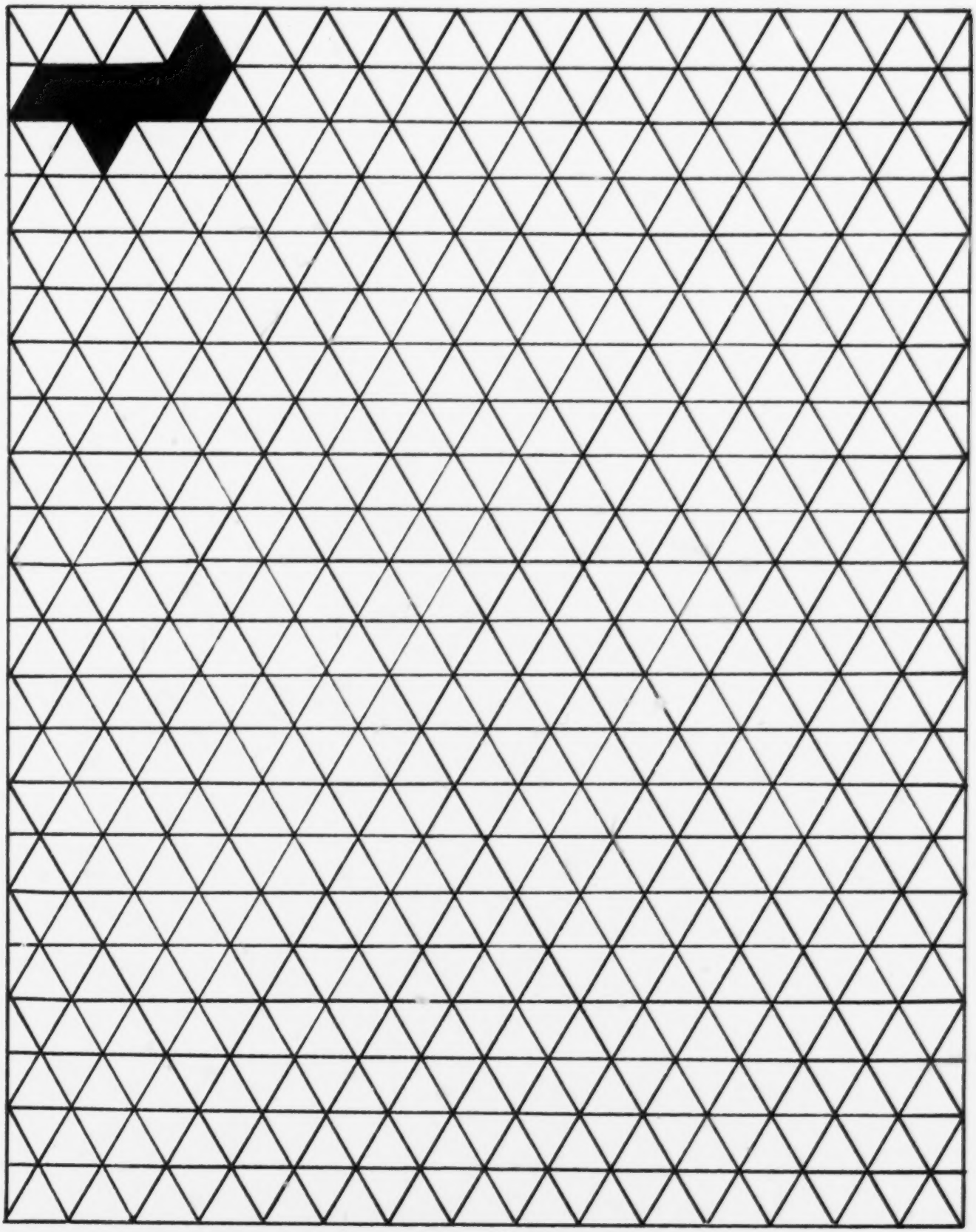


T-A-A

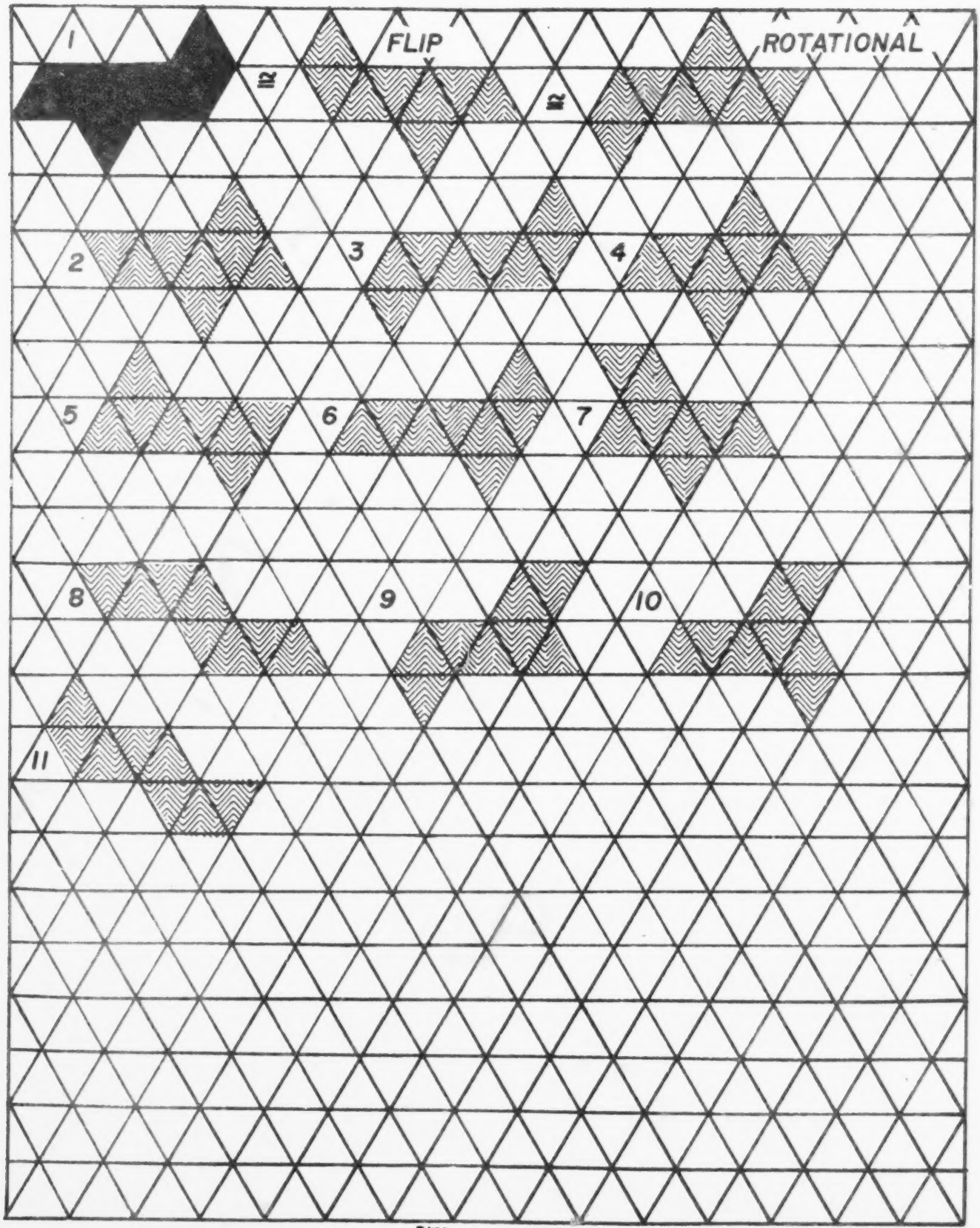
INSERT A-A



INSERT B-B



INSERT B-B



CONTENT AND APPROACH

In number 4 in the students are asked to find different two-dimensional patterns which can be folded to from an octahedron. You may choose to cut down the time spent on this activity since it is similar to part 3. Let each group come up with 5 or 6 and then use transparency G_6 to record different patterns which were discovered by the various groups.

Eleven patterns (other than the original) have been recorded on your insert (T-BB). The students may find some which are not illustrated; these will have to be checked.

Some students may want to further explore this problem on their own.

ON YOUR OWN

The students are asked to work through the On Your Own exercises without the aid of panels and rubber bands. For exercise 1 this should not present any difficulties.

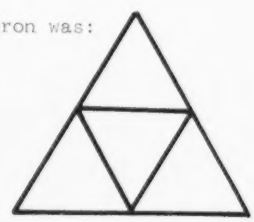
4. See how many different patterns you can find for the octahedron. Record them on the triangular grid provided (insert BB). The original has already been recorded.

   No flips or rotations — please!

ON YOUR OWN

(Try to do the following without the aid of the panels and rubber bands. Fold the patterns in your mind.)

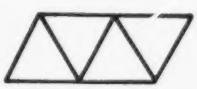
1. The pattern we used for the tetrahedron was:



Could we have used this pattern?

NO

How about this one?



YES

Are there any other patterns which can be used for the tetrahedron? If so, how many?

THERE ARE NO OTHERS.

CONTENT AND APPROACH

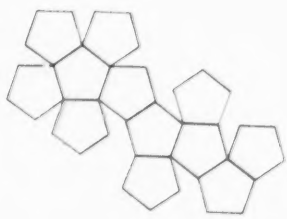
Exercise 2 requires a great deal of concentration in order to fold the illustrated patterns mentally. It may be a good idea to assign only one or two of the three parts to each student.

For parts (a) and (b) the original patterns on page 24 may provide a clue. For the cuboctahedron in part c it may help to notice that each square face is bounded by four triangular faces. Therefore, when a pattern is folded each square must share an edge with four different triangles. Also, no two squares can share an edge. Let the students work on this exercise for awhile. If it produces too much frustration, allow them to use the panels and rubber bands.

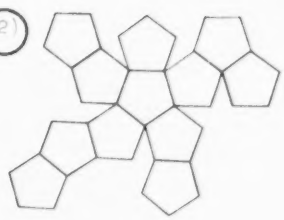
2. For each of the following identify the patterns (from those shown) which you think can be folded to form the given space figure.

a.

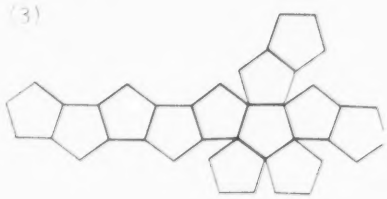
(1)



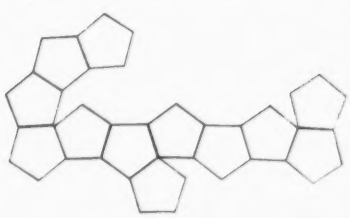
(2)



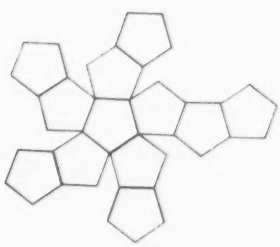
(3)

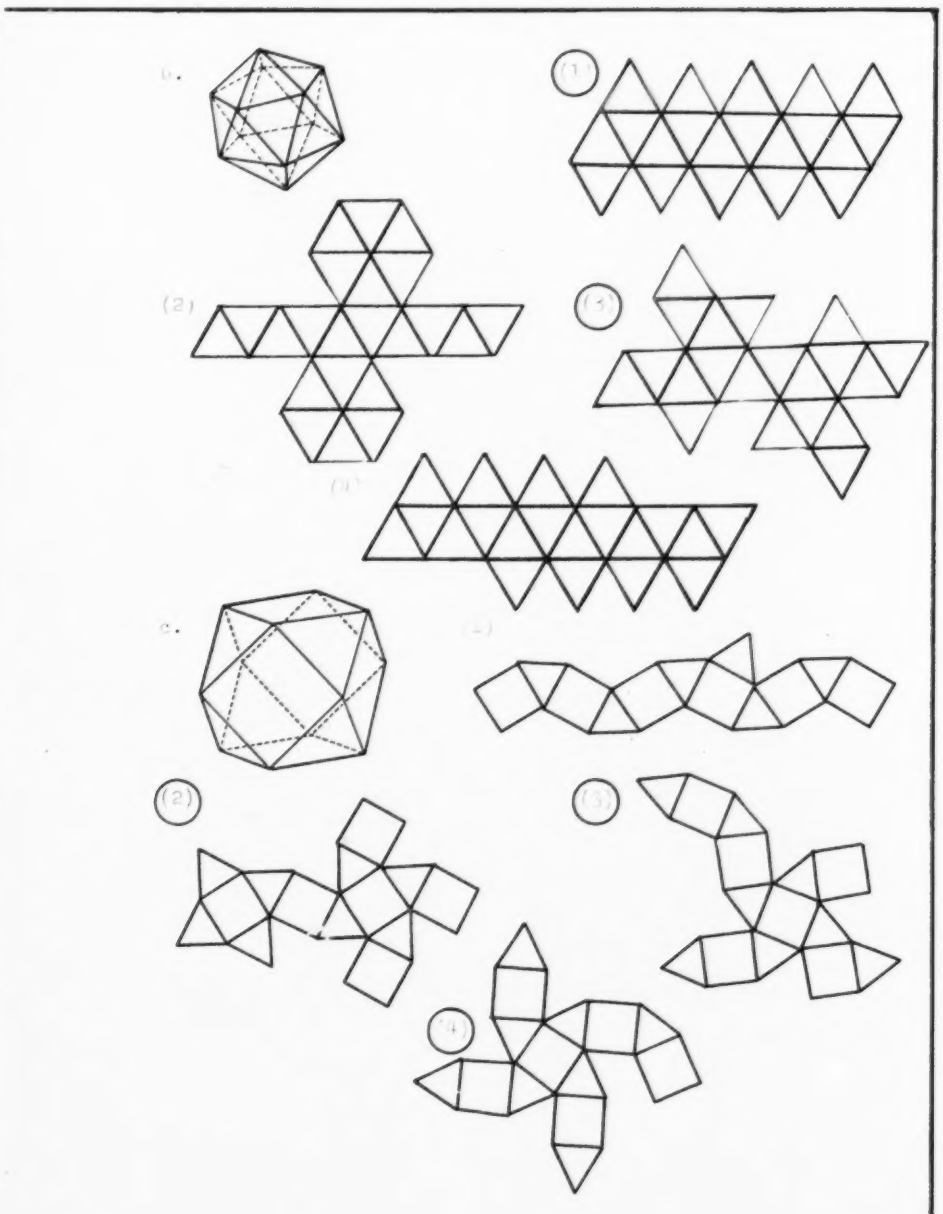


(4)



(5)



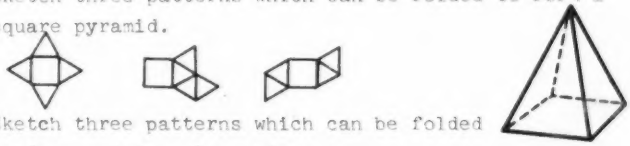


CONTENT AND APPROACH

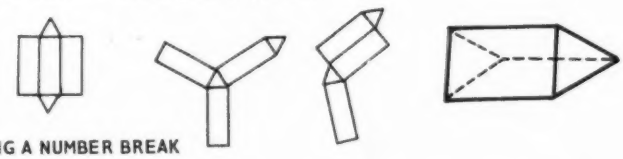
The patterns printed in blue for exercises 3 and 4 are only samples. There are others. You will have to check each student's patterns individually.

This number break (like the first one) does not offer drill on the multiplication and subtraction of whole numbers. Rather it requires that they do some thinking about the process involved in carrying out the operations.

3. Sketch three patterns which can be folded to form a square pyramid.



4. Sketch three patterns which can be folded to form a triangular prism.



TAKING A NUMBER BREAK

Circle the correct response for each of the following.

1. In finding the answer to 25 × 73 the steps could be written as

	73 × 25
Step 1	365
Step 2	146
Step 3	1825

In step 2, 146 is placed where it is to stand for

(a) 2 × 73 (b) (1 × 100) + (4 × 10) + 6
(c) 70 × 25 (d) 20 × 73

2. A student has made the same mistake in each of the following subtraction problems.

57	203	365	518
- 9	- 25	- 87	- 336
59	188	288	282

If he makes the same mistake in the problem $\begin{array}{r} 196 \\ -\ 78 \end{array}$

his answer will be

(a) 122 (b) 128 (c) 118 (d) 274

OBJECTIVES

1. Given a picture of a cube with the middle points marked on each face, the student will be able to identify the space figure which would be formed by connecting adjacent middle points of the cube. ("Octahedron" or "bottom of Traffic Control Tower" are acceptable answers.)
2. Given an octahedron with the middle points marked on each face, the student will be able to identify the space figure which would be formed by connecting adjacent middle points of the octahedron.
3. The student will be able to identify the cube and the octahedron as duals.
4. The student will be able to identify the tetrahedron as the Platonic Solid which is its own dual.

EQUIPMENT AND TEACHING AIDS

1. Models of cube and octahedron from Lesson 3.
2. Scissors (one for each group)
*3. Yarn and needles
*4. Prepared transparencies G_9 - G_{13}
5. Overhead projector and screen
*6. Masters M_2, M_3 (Optional-see page 33); M_1, M_4, M_5 may be useful for the exercises on page 36.

CONTENT AND APPROACH

Lesson 4 deals with a relationship between space figures which is very seldom mentioned in an informal course in geometry. The students will discover that if the middle points of adjacent faces of a cube are connected with line segments an octahedron will appear. If the same procedure is carried out with an octahedron a cube will appear. The cube and the octahedron are known as duals.

The story which precedes the activity is included both for diversion and as a possible (probably not feasible) application of this dual relationship.

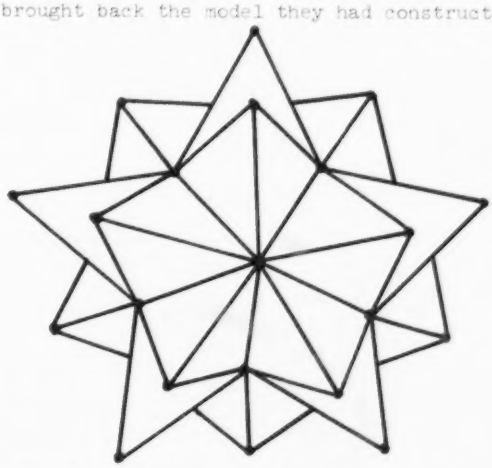
The class activity on pages 34 and 35 is intended to provide a vehicle for discovering the relationship. By using the cardboard models they built in Lesson 3 along with yarn and a needle they can actually see what happens when the middle points of adjacent faces are connected. (PIPE CLEANERS MAY BE USED IN PLACE OF THE YARN AND NEEDLES.)

STRINGING STARS

SPRING FORMAL
DECORATION COMMITTEE MEETING
3:30 ROOM 105

The decorations committee got together to discuss plans for decorating the gym for the Spring Formal. An outer space theme had been selected and the committee was trying to come up with appropriate props.

Judy suggested constructing a huge star polyhedron to hang from the ceiling at the center of the gym. "What's a star polyhedron?" someone asked. Judy ran down to the math room and brought back the model they had constructed from straws.



Everybody liked the idea. They decided to make it out of styrofoam strips and sequins.

Alternate strategy: Instead of conducting the Class Activity for this lesson as it is proposed, you may prefer to make it a pencil and paper activity. Use masters M_2 and M_3 to duplicate copies of the cube and octahedron. Have the students locate the middle points of the figure and connect the points with a colored pen or pencil. This strategy is acceptable. However, it will probably not produce the same degree of comprehension.

CONTENT AND APPROACH

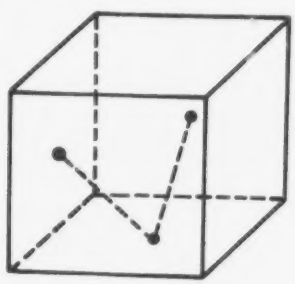
The problem is posed: How will the blinking lights look if they are strung as Dan suggested?

Before the students begin the class activity you might briefly discuss Dan's suggestion. Use the walls of the classroom to demonstrate what Dan means by connecting the middle points on adjoining (adjacent) surfaces. The discussion should only give the students enough information to get them started on the activity. It should not provide a solution.

Some suggestions for conducting the activity:

1. It may be a more efficient use of time to have half of the class work on part 1 of the Class Activity and the other half do part 2. Afterwards, they can share their discoveries with one another.
2. The instructions imply that the students should work in groups of 3. If (as a result of their work in Lesson 3) there is a sufficient number of cube and octahedron models, you may decide to let them work in pairs.
3. After the students have punched a hole through each of the middle points and reassembled their models, suggest that they staple the edges of their model to facilitate the process of connecting adjacent faces with yarn or pipe cleaners. Remind them, however, that one face should be left open so they can see inside the figure.
4. Some students (possibly the girls) will choose to use the needles and yarn; others may choose to use pipe cleaners.

The committee wanted to use some miniature blinking lights to represent stars. They couldn't decide where to string them. Dan suggested placing a hook at the middle of each wall, the floor and the ceiling. He thought the lights could be strung from the hook on a surface to the hook on each of its adjoining surfaces. Dan explained that adjoining surfaces had an edge in common and drew this picture on the board.



Before the group had a chance to discuss Dan's suggestion the bell for the late bus rang and the group hurried out.

CLASS ACTIVITY

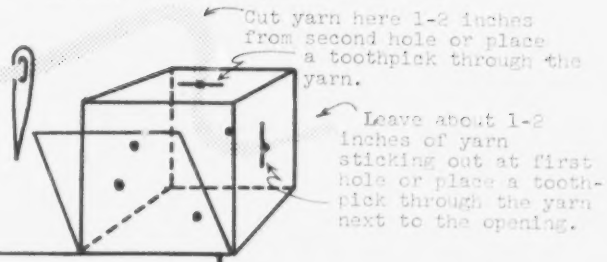
1. You and two of your classmates are members of the decorations committee. The next day in math class you wonder about Dan's suggestion. The gym is cubical. If you string the lights as Dan suggested, what figure would you get?

   A. Use your model of a cube to represent the gym. Take the cube apart and draw the diagonals of each face. The point at which the diagonals meet will be called the middle point. (It is the same distance from each vertex.) Punch a hole at each of the middle points.

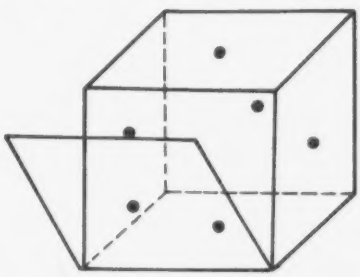
CONTENT AND APPROACH

Demonstrate the following steps for the students who choose to use yarn and needles:

1. Thread a piece of yarn through the eye of the needle. Pull about 3-4 inches of the yarn through the eye so that it doesn't slip out once you start connecting the points.
2. Start outside the model. Pull the needle and yarn through a point on one face. The needle is now in the interior of the cube. Work it through a point on any of the adjacent faces. Be careful not to pull the whole length of the yarn through the first opening or you'll have to start over again. Small pieces of toothpick may be used to keep the yarn from pulling out.


Cut yarn here 1-2 inches from second hole or place a toothpick through the yarn.
Leave about 1-2 inches of yarn sticking out at first hole or place a toothpick through the yarn next to the opening.

B. Put the cube back together: leave one face open.



C. Use yarn or pipe cleaners to represent the lights. Connect the middle points of adjacent faces with the yarn or pipe cleaners until you can recognize the figure being formed.

What figure will be formed if the middle points of adjacent surfaces in a cubical gym are connected with strings of blinking lights?

OCTAHEDRON

2. Suppose the dance is going to be held in a new ultra-modern ballroom shaped like an octahedron. The committee still wants to string lights to represent stars. What will happen if you carry out Dan's suggestion in this situation?
   A. Use your model of an octahedron to represent the room. The middle points have already been marked on the triangular panels. Punch a hole at each middle point of the faces of your octahedron.
   B. Leave one face of the octahedron open.
   C. Repeat step C of Activity 1.

What figure will be formed if the middle points of all adjacent surfaces in an octahedron-shaped room are connected with strings of blinking lights?

CUBE

3. After pulling the yarn through the second point, cut the yarn about 1 or 2 inches from the opening or place a toothpick through the yarn close to the opening. (This will eliminate cutting the yarn each time.)
4. Connect this second point with a point on one of its adjacent faces. Be careful not to break the original connection. The use of toothpicks as stoppers will make the process much easier.

Note: You may notice that some students are more adept at carrying out this activity and finish quickly. If they are interested, let them connect the points on adjacent faces on the cardboard models of the dodecahedron and icosahedron. These will come in handy for demonstration purposes if the class has trouble answering questions 6 and 7 on page 36.

CONTENT AND APPROACH

Transparencies G_9 through G_{13} should be used in conjunction with these exercises. However, don't spoil the discovery aspect by using them prematurely.

These exercises introduce the students to the term dual. Two space figures are duals if connecting the middle points of adjacent faces of one produces the other and vice versa.

The students should not have any difficulty answering questions 1 and 2. Exercises 3 and 4 should also be relatively easy to answer. The tetrahedron is the only Platonic Solid which is its own dual.

They may find it difficult to see the dual relationship that exists between the dodecahedron and icosahedron. Give them time to try answering 6 and 7. If they are having trouble use transparencies G_{11} and G_{12} to provide some help.

In exercise 9 they should discover that the a square pyramid is its own dual. Something to think about — Is every pyramid its own dual?

Their work with the dual relationship in this lesson should carry over into Lesson 5 where they will count the number of faces, edges and vertices of various polyhedra. After they have completed the table on page 38 they should examine their entries in the faces and vertices columns for the figures which are duals. This observation will help them see why the dual relationship exists. The number of vertices of one figure is the same of the number of faces for its dual and vice versa.

ON YOUR OWN

1. What figure was formed by connecting the middle points (of adjacent faces) of a cube? **OCTAHEDRON**

2. What figure was formed by connecting the middle points of an octahedron? **CUBE**

3. What figure would be formed by connecting the middle points of a tetrahedron? **TETRAHEDRON**

4. Two Platonic solids are called duals if one has the same number of vertices as the other has faces and vice versa. Which Platonic solid is its own dual? **TETRAHEDRON**

5. What is the dual of the cube? **OCTAHEDRON**

6. Without actually connecting them, identify the kind of figure that would be formed by connecting the middle points of an icosahedron.

DODECAHEDRON

7. What kind of figure do you think would be formed by connecting the middle points of a dodecahedron?

ICOSAHEDRON

8. What is the dual of the dodecahedron? **ICOSAHEDRON**

9. Is there a figure which is the dual of a square pyramid? If so, what is it? **SQUARE PYRAMID**

OBJECTIVES

1. Given the straw models of the buildings of Geopolis, the student will be able to count the faces, edges and vertices of the models.
2. Given a table listing the number of faces, edges and vertices of various polyhedra, the students will be able to discover the relationship $F + V = E + 2$.
3. Given some space figures (which are not polyhedra) the student will discover that these space figures do not satisfy the general form of Euler's relation ($F + V = E + 2$).

EQUIPMENT AND TEACHING AIDS

1. Straw models from Lesson 1 (for reference)
2. Models for exercises 8 – 13 on pages 40 – 42 (It is strongly suggested that models of some or all of these figures be made available to the students.)
3. Prepared transparency G_{14}
4. Overhead Projector and screen

CONTENT AND APPROACH

In Lesson 4, the students discovered a relationship between pairs of polyhedra; in Lesson 5 they will be led to discover a relationship between a polyhedron and the number of faces, edges and vertices (F + V = E + 2). Euler (pronounced Oiler) discovered the relationship in the 18th century and thus it bears his name.

Let the students discover the formula on their own. The exercises on page 29 will help them in their discovery.

THINGS TO DISCUSS

What are some of the other contributions made by Leonard Euler to the development of mathematics?

What physical handicap made his contributions to mathematics even more amazing?

FACES, EDGES, VERTICES - DO THEY ADD UP?

In the 18th century, Leonard Euler, a Swiss mathematician, discovered a relationship between the number of faces, vertices and edges of a polyhedron.

Fill in the table on the next page and see if you can discover the same relationship that Euler found.

CONTENT AND APPROACH

Give the students time to fill out the table. Let them work together. You might pass the straw models around the classroom. This will make counting easier for them. They can also refer to pages 2, 3, and 13 for pictures of these polyhedra.

After they have had time to fill out the table on their own they can help you complete the blank table on G_{14}. This will give them a chance to check their answers.

Part 8 of the table will probably present some difficulties. You might have the entire class complete 1 through 7, then either do 8 together or let the students who finish first tackle number 8.

STRUCTURE	NUMBER OF FACES	NUMBER OF VERTICES	NUMBER OF EDGES
1. Bottom of the Traffic Control Tower (Octahedron)	8	6	12
2. Top of the Traffic Control Tower (Tetrahedron)	4	4	6
3. Electronic Servant Rental (Cube) (outer structure)	6	8	12
4. Cloud Shooting Station (Dodecahedron)	12	20	30
5. Nuclear Power Chamber (Icosahedron)	20	12	30
6. Geopolis Hilton (Cuboctahedron)	14	12	24
7. Middle structure of Under H_2O Research Center (Hexagonal Prism)	8	12	18
8. Geopolis State University (Star polyhedron)	60	32	90

CONTENT AND APPROACH

The first three exercises in this set should provide a clue for discovering Euler's Formula. If these examples are not sufficient to uncover the relationship, they can go back to the table and for each figure compare the sum of the numbers in the first two columns (FACES, VERTICES) with the number in third column (EDGES).

EXERCISES

1. For the tetrahedron:

   number of faces + number of vertices = 8
   number of edges = 6

2. For the octahedron:

   number of faces + number of vertices = 14
   number of edges = 12

3. For the dodecahedron:

   number of faces + number of vertices = 32
   number of edges = 30

4. Write a sentence which expresses the relationship between the number of faces, vertices and edges for the figures in the above exercises. [Hint: Study the two answers that you have recorded in each exercise above. If that doesn't provide a clue, compare the numbers in the first two columns (Faces, Vertices) of the table with the numbers in the third column (Edges).] **FACES + VERTICES = EDGES**

   OR FACES + VERTICES - 2 = EDGES

5. Which of the other figures listed in the table follow the same pattern?

   ALL OF THE FIGURES

6. A certain polyhedron has 12 vertices and 5 faces. How many edges does it have?

   15

7. A certain polyhedron has 500 vertices and 1000 edges. How many faces does it have?

   FACES + 500 = 1,000 + 2 ∴ FACES = 502

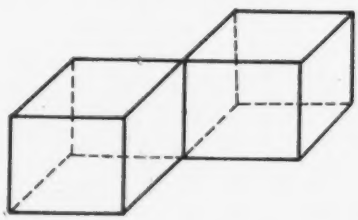
CONTENT AND APPROACH

For exercises 8 and 9 the General Form of Euler's Formula does not hold.

Explanation: (a) For the figure in exercise 8 – We know that for each cube separately $F + V = E + 2$. For both cubes then $2F + 2V = 2E + 4$. However, they share an edge and two vertices $2F + (2V - 2) = (2E - 1) + 4$ or $F + V = E + 3$

(b) For the figure in exercise 9 – For each square pyramid $F + V = E + 2$ For both $2F + 2V = 2E + 4$. However, they share a vertex so $2F + (2V - 1) = 2E + 4$ or $2F + 2V = 2E + 3$ or $F + V = E + 3$

8. Consider the following figure formed by two cubes which share an edge:



number of faces = 12
number of edges = 23
number of vertices = 14
F + V = E + 3

Is the result different for this figure? YES If so, why?

SEE COMMENTS

9. Consider the following figure formed by two square pyramids which share a vertex:

number of faces = 10
number of vertices = 9
number of edges = 16

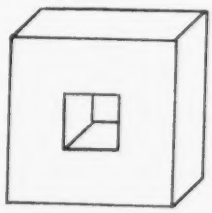
Does Euler's formula (F + V = E + 2) hold for this figure? Why or why not? NO

SEE COMMENTS

CONTENT AND APPROACH

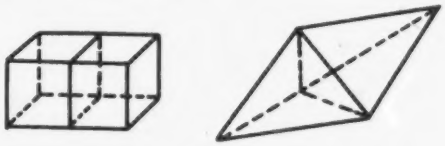
Exercises 8 and 9 may cause the students to conclude that the two figures in exercise 10 will not satisfy Euler's formula. However, counting will establish that for both figures F + V = E + 2. If one were to consider the face which is "shared" as a distinct face of the figure then F + V = E + 3. If we agree that these figures represent closed surfaces, it would not be possible to consider the "shared" face as a face of the figure.

For the figure in exercise 11, the students are instructed to count all the faces, edges and vertices. (This means counting all the visible faces, edges and vertices of each sugar cube.) In this special case, Euler's Formula will not hold. If we disregard the faces, vertices and edges of each small cube and look at the figure in the following way,



then F = 10, V = 16 and E = 24 so F + V = E + 2.

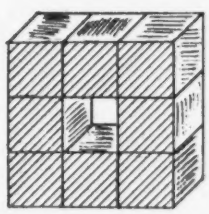
10. Consider the following figures formed by two regular polyhedra which share a face:



Do you think that Euler's formula will hold for these figures? _____

Explain Your Answer **SEE COMMENTS**

11. Study the following figure which is made up of eight sugar cubes.



If you count all the faces, edges and vertices which are in the figure, does Euler's Formula hold? (Hint: There are 32 faces in the figure.)

faces = **32**
vertices = **32**
edges = **64**

F + V = E + **0** **SEE COMMENTS**

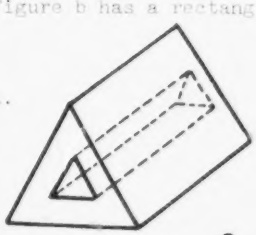
CONTENT AND APPROACH

The students will find that for both figures in exercise 12, F + V = E + 2. Not all polyhedra with "holes" satisfy this general form of Euler's Formula. A good extension activity would consist of finding one that does not. (Hint: Find one with more than one hole.)

After working through exercises 1-12 the students should be able to answer exercise 13 without counting. Caution the students that the two hexagonal pyramids in e. share only an edge.

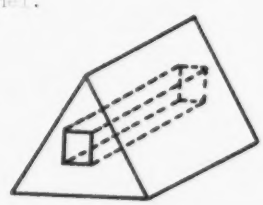
12. The two triangular prisms pictured below have tunnels cut out of them. Figure a has a triangular tunnel; figure b has a rectangular tunnel.

a.



number of faces = 8
number of vertices = 12
number of edges = 18
F + V = E + 2

b.

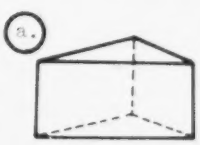


number of faces = 9
number of vertices = 14
number of edges = 21
F + V = E + 2

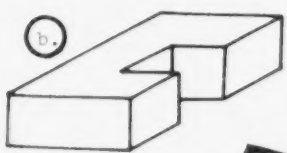
Do both figures satisfy Euler's Formula? YES If not, why not? SEE COMMENTS

13. Which of the following figures do you think will satisfy Euler's Formula? Try to answer without counting.

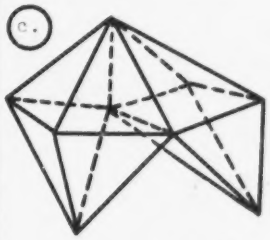
a.



b.

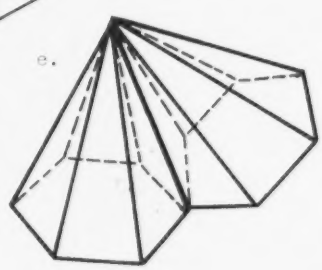


c.



d.

e.



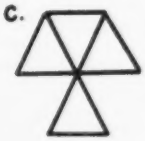
f.

✔ POINT

1. Which of the following patterns can be folded to form a tetrahedron? b,d

a.

b.

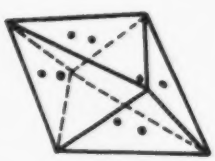
c. 

d.

2. Sketch three patterns which can be folded to form a cube.

SEE T-AA

3. Given the following octahedron with the middle points of each face indicated:



The figure formed by connecting the middle points of adjacent faces is a: a

a. cube. b. tetrahedron. c. octahedron. d. pyramid.

4. The dual of an octahedron is a: a

a. cube. b. tetrahedron. c. icosahedron. d. octahedron.

5. If a certain polyhedron has 11 faces and 15 vertices, it has d edges.

a. 26 b. 52 c. 28 d. 24

6. The smallest number of vertices a polyhedron can have is 4.

CONTENT AND APPROACH

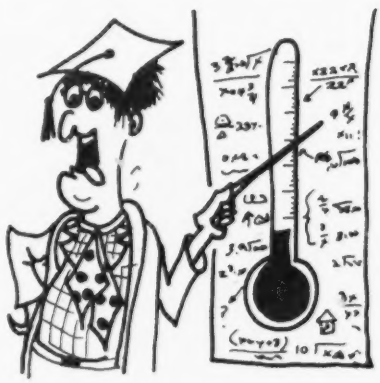
Lesson 6 presents a dressed-up version of a commonly used space perception puzzle: A cube is painted red on all faces and cut into 27 smaller cubes. If you take the cube apart, how many of the smaller cubes will have three red faces, two red faces, one red face, no red faces?

The story was written for the students, in their language. It gives them a chance to develop their space perception and also to do some problem solving. It may also be considered a diversion.

Ask the students to read the story and study Plat's crumpled sheet of paper on page 46. To avoid some confusion, you might emphasize that the secret formula ("TPS") is first put into small sugar cubes and then these small cubes must be packed in a larger cubical structure.

DOTS, CUBES AND DRAGSTERS

A call came into the Sugar Lake Police Station. The voice quivered, "It's been stolen: it's been stolen." Sargeant Sunday tried to calm the man and get the lowdown. Finally, the man identified himself. It was Professor Plat Formate. A cube-shaped package containing his secret formula, "TPS", had been lifted from his laboratory. If the cube-shaped structure made up of purple dotted sugar cubes containing "TPS" was not recovered within two hours the annual drag race at the Sugarflats would have to be cancelled. Sunday knew instantly that this was a case for the "Rod Squad." The squad was contacted and within seconds their wheels were rolling and the pair began a search for the "TPS".



Meanwhile in a television press conference the Professor explained that there was enough "TPS" in the cube-shaped package for all nine of the heaps that were to run at the Flats. If the thief should use the entire package he would burn out his piston rings.

LESSON 6

OBJECTIVES

1. Given an $n \times n \times n$ ($n = 2, 3, 4$) cube made up of sugar cubes with a dot on each of the outer faces, the student should be able to answer the following:

   a. How many single sugar cubes are needed to build the large cube?

   b. How many cubes have five dots? three dots? two dots? one dot? no dots?

2. For any $n \times n \times n$ cube made up of sugar cubes with a dot on each of the outer faces, the student will be able to answer the following:

   How many cubes have six dots? three dots? two dots?

3. Given sugar cubes and the instruction: "Find all possible (different) arrangements of not more than four cubes such that each arrangement has at least one arm," the student will be able to find the seven possible arrangements.

EQUIPMENT AND TEACHING AIDS

*1. Sugar cubes (packets of 30)

2. Elmer's glue

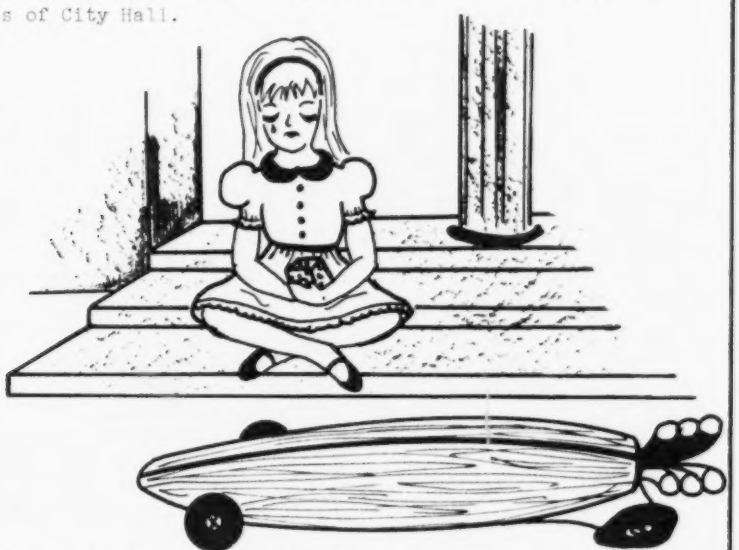
*3. Prepared transparencies G_{16}, G_{16}

4. Overhead projector and screen

CONTENT AND APPROACH

Please observe the students' reactions to this story. Record them on this page.

While the Professor was making his television debut, the "Rod Squad" received a tip concerning the possible whereabouts of the thief. They raced to the center of town where they spotted a small frail girl crying as she was sitting on the steps of City Hall.



She was holding a cube-shaped package. One of the slicks on her motorized surfboard was as flat as the board. She was foiled in her attempt to have the fastest board on wheels. She surrendered the cube-shaped structure containing purple dotted sugar cubes, without any resistance.

The squad raced to the studio to pick up the Professor. As the groups burned down Main Street, on their way to the Flats, one of the "Rod Squad" asked Plat why he had stored the formula in the purple dotted cube-shaped package. Plat pulled a crumpled sheet of paper from his back pocket and passed it to him.

CONTENT AND APPROACH

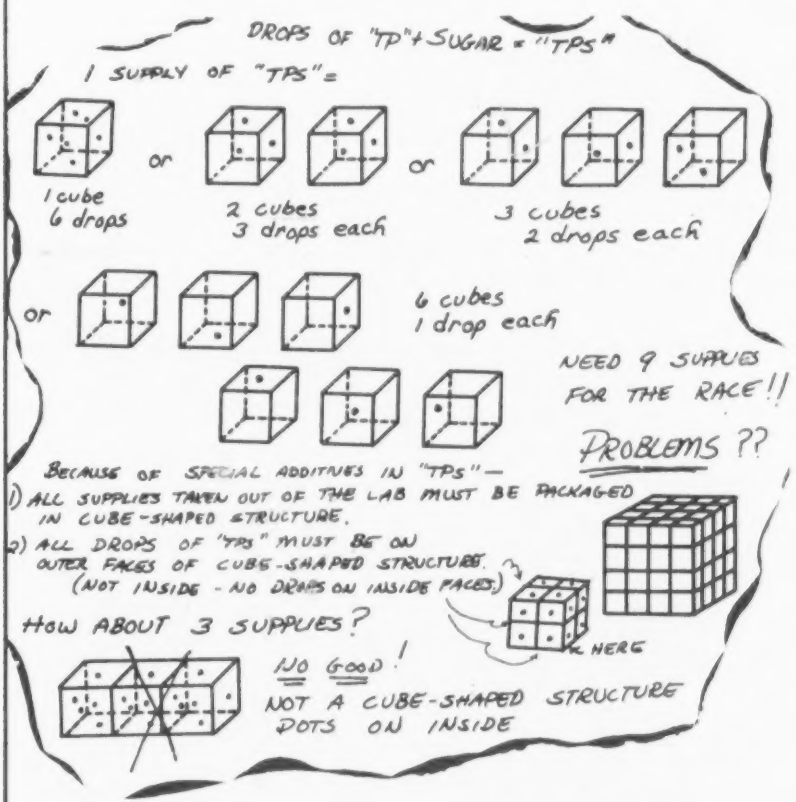
After the students have had a chance to examine Plat's scribbles, use G_{15} to discuss the questions at the top of the page.

ANSWERS

1. Drops can be stored in the sugar cubes in four different ways — 1 cube with six drops; 2 cubes with 3 drops on each, etc.
2. Cubes must be packaged in a cube-shaped structure because of special additives in "TPS".
3. After the sugar cubes have been arranged in a cube-shaped structure, all drops must be visible. They must be on the outer faces of the structure.

As you study Plat's scribbles keep these questions in mind:

1. What are the different ways that drops can be stored in sugar cubes to give one supply of "TPS"?
2. Why must the sugar cubes be packaged in a cube-shaped structure?
3. When the sugar cubes are arranged in a cube-shaped structure, must all the drops be visible?


DROPS OF "TP" + SUGAR = "TPS"
1 SUPPLY OF "TPS" =
or
or
1 cube
6 drops
2 cubes
3 drops each
3 cubes
2 drops each
or
6 cubes
1 drop each
NEED 9 SUPPLIES FOR THE RACE!!
PROBLEMS ??
BECAUSE OF SPECIAL ADDITIVES IN "TPS" —
1) ALL SUPPLIES TAKEN OUT OF THE LAB MUST BE PACKAGED IN CUBE-SHAPED STRUCTURE.
2) ALL DROPS OF "TPS" MUST BE ON OUTER FACES OF CUBE-SHAPED STRUCTURE. (NOT INSIDE - NO DROPS ON INSIDE FACES)
HOW ABOUT 3 SUPPLIES?
HERE
NO GOOD!
NOT A CUBE-SHAPED STRUCTURE
DOTS ON INSIDE

CONTENT AND APPROACH

Each student should receive a plastic bag containing thirty sugar cubes. They will need them for the exercises and for the class activity. They will only use 27 for the activity so they can afford to eat a couple.

Let the students work the exercises together. They can pool their cubes for the exercises as long as they get 27 back before the class activity.

ANSWERS

4. Some students will get the answer 9 by figuring 9 dots on a face of the large cube, six faces, therefore 54 dots or 9 supplies.

   Others will take the large cube apart and see:

   8 cubes with 3 drops or 4 supplies
   12 cubes with 2 drops or 4 supplies
   6 cubes with 1 drop or 1 supply

   All together 9 supplies.

This second method was the intended one and should be mentioned when going through the exercises.

The story you have just read is false (or at least we think it is). The names have been changed (just in case) to protect the innocent. Use the information from Plat's scribbles to work the following exercises.

EXERCISES

(Your teacher will give you a supply of sugar cubes to use in working the following exercises.)

1. If a single sugar cube with a drop on each face holds one supply of "TPS", why didn't the Professor simply package nine of these single cubes for the race?

   NINE CUBES CANNOT BE STORED IN CUBICAL SHAPE. EVEN IF THEY COULD BE, ALL DROPS WOULD NOT BE ON OUTER FACES.

2. If the sugar cubes were shaped into a 2 × 2 × 2 cube-shaped structure (2 cubes on a side) with drops on each of the outer faces, how many cars could have received a supply of "TPS"?

   4

3. Build a 3 × 3 × 3 cube:
   a. How many cubes are needed? **27**
   b. If a drop is placed on each of the outer faces, how many cubes contain:
      three drops? **8**
      two drops? **12**
      one drop? **6**
   c. Do all the cubes which make up the 3 × 3 × 3 cube have at least one drop?

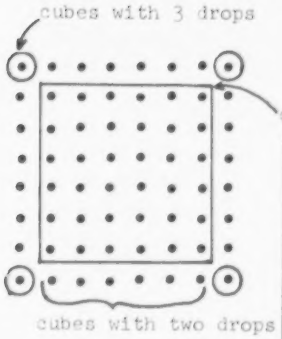
      NO, I HAS NO DROPS.

4. How many supplies of "TPS" can be stored in a 3 × 3 × 3 cube? **9**

SEE COMMENTS

CONTENT AND APPROACH

The students should realize that for any $n \times n \times n$ cube no single cube will have more than 3 drops. For exercises 6 and 7 suggest that they consider an array of drops similar to those which they would see on one face of the cube. For an $8 \times 8 \times 8$ cube:



For any $n \times n \times n$ cube there are 8 cubes with three drops. For this $8 \times 8 \times 8$ cube there are $n - 2$ or 6 cubes along each edge which would have 2 drops. There are 12 edges so 12×6 or 72 cubes have 2 drops. For the $8 \times 8 \times 8$ cube we can see that there are 6×6 or 36 cubes on each face which would have one drop. There are six faces so 6×36 or 216 cubes have one drop. To find the number of cubes with no drops — Total number of cubes in structure is $8 \times 8 \times 8$ or 512. $512 - (8 + 72 + 216) = 216$

Optional Activity

Have the students work with other examples of $n \times n \times n$ cubes where n is greater than 1. See if they can discover that:

(1) The number of cubes with 3 drops will always be 8.

(2) The number of cubes with 2 drops will always be $12 \times (n - 2)$.

(3) The number of cubes with 1 drop will always be $6 \times (n - 2)^2$.

(4) The number of cubes with no drops will always be $(n - 2)^3$.

5. If the sugar cubes are packaged in $4 \times 4 \times 4$ cube,
   a. How many single cubes are needed to build it? 64
   b. How many cubes would have four drops? NONE
      three drops? 8
      two drops? 24
      one drop? 24
      no drops? 8

6. In an $8 \times 8 \times 8$ cube-shaped structure how many cubes would have six drops? NONE
   five drops? NONE
   two drops? 72
   three drops? 8
   one drop? 216

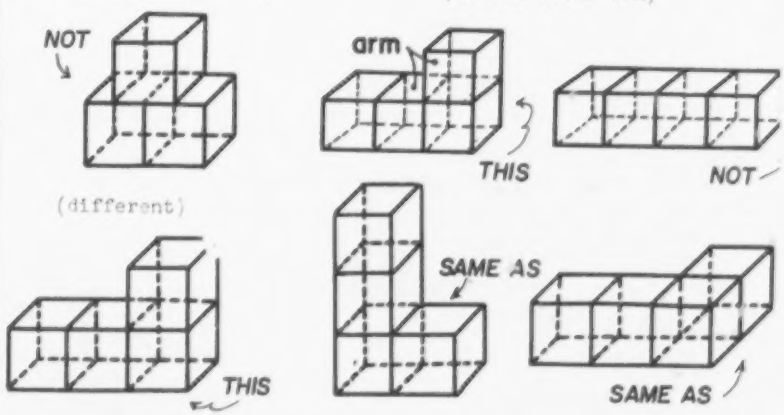
7. In a $100 \times 100 \times 100$ cube-shaped package how many cubes would have:
   four drops? NONE
   two drops? 1176
   three drops? 8
   one drop? 57,624

8. If nine supplies of "TPS" were needed for the race, the cube-shaped package stolen from the Professor's lab was a:
   a. $2 \times 2 \times 2$ cube (b.) $3 \times 3 \times 3$ cube c. $4 \times 4 \times 4$ cub
   d. $6 \times 6 \times 6$ cube e. $8 \times 8 \times 8$ cube f. $9 \times 9 \times 9$ cub

CLASS ACTIVITY

1. Use your cubes to form all possible <u>different</u> arrangements of not more than four cubes. In each arrangement the cubes should meet face on face; there should be at least one arm.

(meet face on face) (at least one arm)

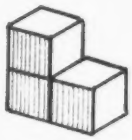


CONTENT AND APPROACH

As the students work through the Class Activity they should keep in mind that the arrangements must: (See illustrations at bottom of page 48.)

1. meet face on face.
2. have at least one arm. (They can have more than one.)
3. be different (no flips or rotations allowed).

You might suggest that they start with the only piece which is made up of three cubes.



They can build the other pieces from this one.

The students should find seven different arrangements. See Transparency G_{16} for the solution. They should check their arrangements with you before they glue them together.

The problem posed in For The Puzzle Enthusiast is difficult. It is said that there are only two ways to arrange these six pieces to form a $3 \times 3 \times 3$ cube. Interested students may want to experiment with this in their spare time.

When you think you have found all of them, check with your teacher. (Hint, there are more than 5.)

2. Glue the cubes together.

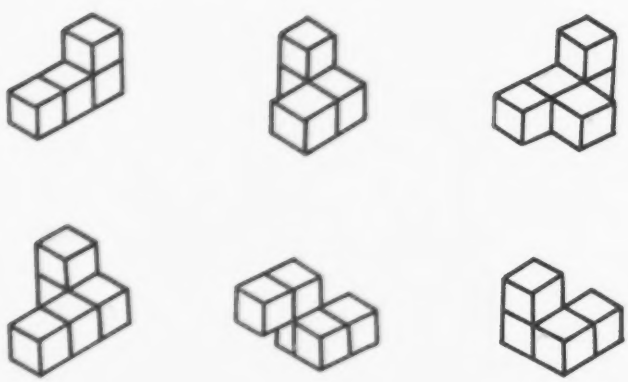
3. See if you can put the different pieces together in one $3 \times 3 \times 3$ cube.

4. Can you find another way to fit them together?

FOR THE PUZZLE ENTHUSIAST

If you enjoyed working the above puzzle and you have some patience left try this: (Get 27 more cubes from your teacher.)

1. Arrange the cubes into six pieces.



2. Build a $3 \times 3 \times 3$ cube with these six pieces.

CONTENT AND APPROACH

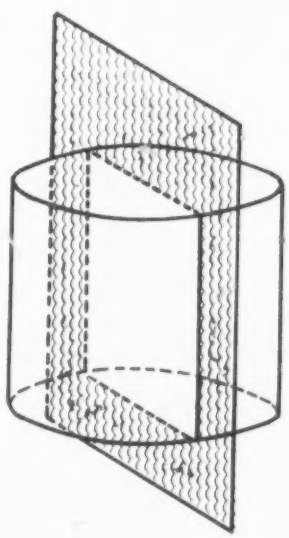
Slicing Through Solids deals with an important concept of non-metric geometry - the intersection of a plane with a space figure. In a laboratory situation, the students will actually slice through clay models to see the intersections which would be made by a plane.

The introduction in this lesson portrays a "real world" example of what happens when a plane (represented by the wall) intersects a space figure (building).

SLICING THROUGH SOLIDS

The chief construction engineer of the new cylindrical-shaped computer building at Apollo Lift-Off Center is faced with a serious problem. A glass partition is to run through the middle of the building and extend from the roof to the ground floor. The computer will be housed in half of the building and kept in super-cold temperatures. Computer operators will work on the other side of the glass wall and keep a close watch on the electronic wizard.

This picture was drawn by the construction engineer to help him visualize what shape the wall must be.



It is important that he order a partition with the correct shape. A mistake would be costly.

OBJECTIVES

1. Given a set of pictures showing a plane intersecting a cone and a cube at various angles the student will be able to:
    a) slice through a clay model of a cone and a cube to show the cuts that the plane would make;
    b) identify the shape of the cut.
2. Given a picture showing a sphere with planes intersecting it at various angles, the student will be able to identify the various cuts as circular in shape.
3. Given a picture of two cylinders which are the same size and shape and a plane passing through both cylinders parallel to their bases, the student will be able to identify the cuts made on both cylinders as:
    a) circular in shape;
    b) the same size and shape;
    c) the same size and shape as the bases of the cylinders.
4. Given a picture showing any polyhedron with a plane intersecting it, the student will be able to identify the shape of the cut.

EQUIPMENT AND TEACHING AIDS

*1. Clay
*2. Cardboard patterns for a cube and cone
*3. Cardboard cylinders
*4. Strips of wire
5. Scissors
6. Masking tape

CONTENT AND APPROACH

The discussion questions start the students thinking about what happens when a two-dimensional object (plane) is passed through a three-dimensional object (cylindrical building).

They should realize that the wall will be rectangular in shape.

Question 2 asks them to think about a plane intersecting the building at a different place. The shape of the wall will still be rectangular but the dimensions will be different.

In question 3 — if the building was cone-shaped the glass partition would have to be triangular-shaped.

The paragraph at the bottom of the page introduces the idea of a plane. You might ask them for some other physical representations of a plane.

THINK ABOUT IT

1. You are a rookie engineer on the construction job. You want to help your boss out. What shape wall would you suggest he order?

2. Suppose that the new computer turned out to be smaller than expected. The glass partition must still run from the roof to the ground floor but it will only cut off about one-third of the building. New plans must be sent to the glass company immediately. What will be the shape of this partition?
   Will this partition be the same size as the first one that was ordered?

3. If the building were shaped like a cone instead of a cylinder, what would be the shape of the glass partition (assuming that the partition runs from the tip of the cone to the base)?

The following experiments will help you see what happens when a two-dimensional surface passes through a three-dimensional solid. The glass partition in the computer building is a two-dimensional surface, and it represents part of what mathematicians call a <u>plane</u>. A plane can be thought of as a flat surface with no thickness, extending forever in two directions (length and width). Some common objects which represent parts of planes are the top of your desk, a flat sheet of paper, the surface of a calm lake, a thin sheet of ice, and the wall of your classroom. We will call flat two-dimensional surfaces <u>portions</u> of planes.

CONTENT AND APPROACH

The students should work in groups of 2 or 3 to carry out the experiments in this lesson. You can set up the different stations around the room or you can supply each group with all the materials needed for the four stations.

Stations 1, 2 and 4 call for the use of cardboard models to form the clay solids. You may prefer to have the students work the clay into the shapes without using the cardboard molds.

They are instructed to use wires to cut through the clay models. You have received a roll of wire which is not yet cut. Cut the wire into 14 inch strips. Fold two pieces of masking tape about three inches from each end of the cut wire. This will enable them to wrap the 3 inches at each end around their index fingers and hold on to the masking tape as they slice through the model.

NOTE: If the station calls for multiple slices of the same clay figure it is important that the students put the model back together before making a new slice. If they fail to do this, they will get shapes other than the ones intended.

STATION 1

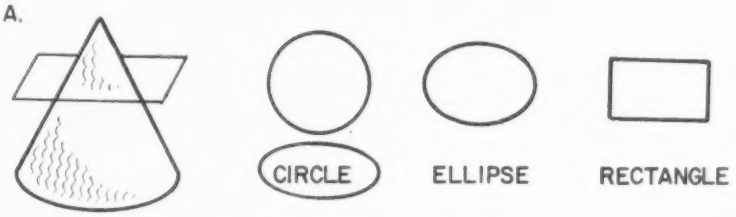
Materials: cone pattern, clay, wire, tape.

1. Fold cardboard pattern so that A is next to B. Tape the edges together.
2. Pack the clay into the cardboard cone, making sure it goes down the tip of the cone.
3. Release the pattern from the clay model.

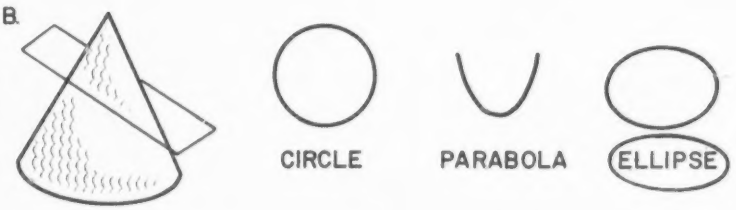
Use your wire to slice through the clay model in order to show the cut that would be made if a plane passed through the cone in each of the five different ways. Some possible shapes for each cut are shown. Circle the name of the shape which looks most like the one you get.

(Put the cone back together before making a new cut.)

A.



B.



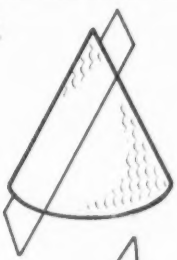
CONTENT AND APPROACH

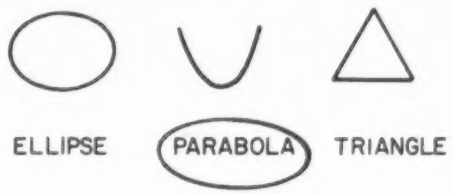
The shapes which the students will get by slicing through the clay model of a cone represent the shapes which are known as the Conic Sections. These shapes have many useful and interesting properties which your students may wish to explore on their own.

THINGS TO DISCUSS

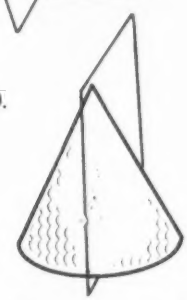
How can you describe the intersection of a plane with the tip (vertex) of a cone.

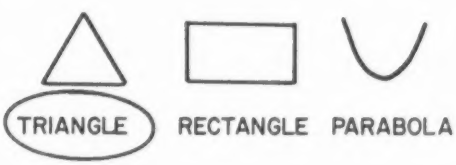
C.





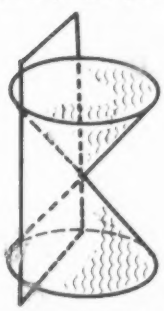
D.

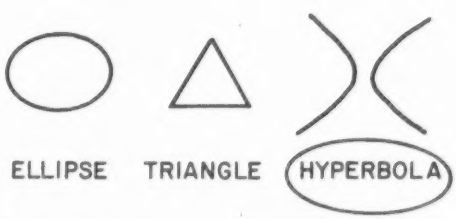




What if you had two clay cones intersecting at a point with a plane passing through both of the cones?

E.



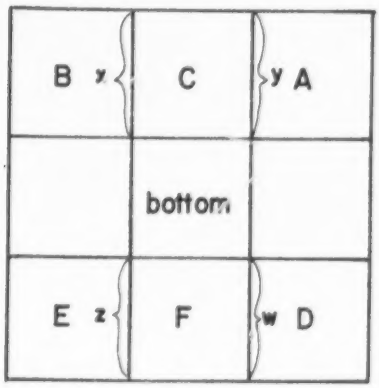


The circle, ellipse, parabola and hyperbola belong to a special set of plane figures called Conic Sections. These curves have some interesting properties which we will look at in Geometric Excursions II.

CONTENT AND APPROACH

The materials for Station 2 include a tagboard cube. You have received tagboard patterns which will have to be folded to form a cube. The following are instructions for forming the cardboard mold for the cube:

1. Cut out the 3 × 3 square
2. Cut segments x, y, z, w
3. Crease all the other lines in the pattern
4. Fold flap C up and fold flaps B and A behind C. Do the same for flaps F, E and D.
5. Tape the outside edges of the model.



STATION 2

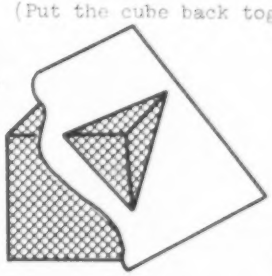
Materials: tagboard cube, clay, wire

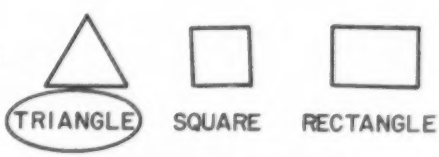
1. Pack clay into cube. (tightly)
2. Release tagboard from clay model.

Use your wire to slice through the clay model to show the cut that would be made if a plane passed through the cube in each of the five different ways. Some possible shapes for each cut are shown. Circle the name of the shape which looks most like the one you get.

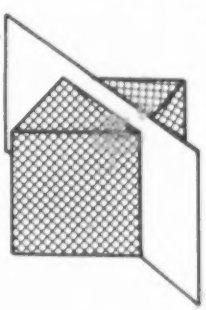
(Put the cube back together before making a new cut.)

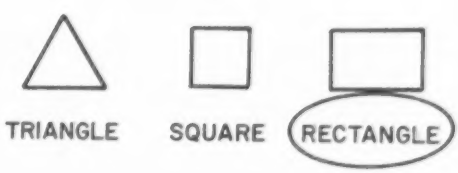
A.





B.





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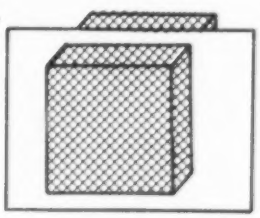
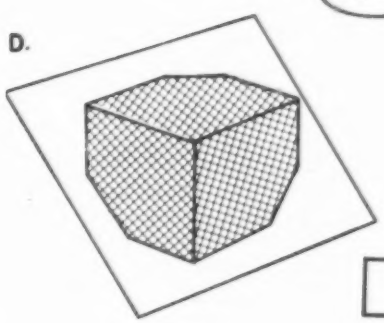


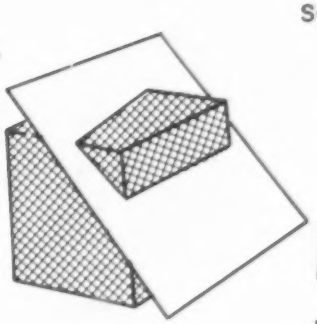
SQUARE RECTANGLE TRAPEZOID

D.



SQUARE HEXAGON PENTAGON

E.



RECTANGLE TRIANGLE TRAPEZOID

STATION 3

Materials: clay, wire

1. Roll the clay into a ball.
2. Slice through the clay model in at least four different ways. (Put the sphere back together before making a new slice.)

Sketch the shape of each cut.

1. 2.

EACH CUT SHOULD BE CIRCULAR IN SHAPE.

3. 4.

What do all four shapes have in common?

THEY ARE ALL CIRCULAR IN SHAPE.

Are all four shapes the same size?

NO

What would be the shape of the largest possible cut? What would you have to do to get this cut?

CIRCLE; SLICE THROUGH THE CENTER OF THE SPHERE.

CONTENT AND APPROACH

STATION 4

Materials: clay, two paper cylinders, wire.

1. Pack clay tightly into both paper cylinders.

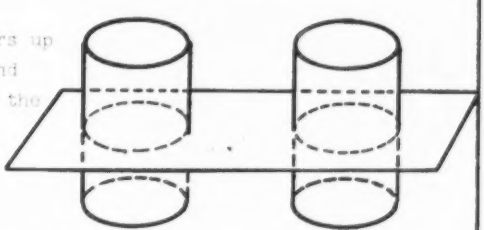
2. Release paper tubes from clay models. (Make sure the models are the same size.)

3. Trace the bottom and top face of each of the cylinders.

What can you say about their shape and size?

SAME SHAPE AND SIZE (APPROX.); CIRCULAR IN SHAPE.

4. Stand the two cylinders up next to one another and slice through both at the same time.



How do the plane sections that you get compare with each other in size and shape?

SAME SHAPE AND SIZE

How do each of the plane sections compare with the top and bottom bases of the cylinder it was cut from?

SAME SHAPE AND SIZE AS THE BASES.

Name or sketch at least three different shapes that a plane could cut if it were passed through a cylinder.

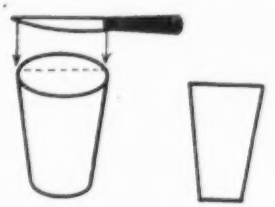
CONTENT AND APPROACH

The On Your Own exercises are an extension of the station activities. The students have been given the opportunity to slice through clay models and see the shapes which result from making various cuts. In the exercises they should use this experience and their imagination to identify shapes that would result if they were to slice through various three-dimensional objects. The arrows indicate the direction of the cut to be made.

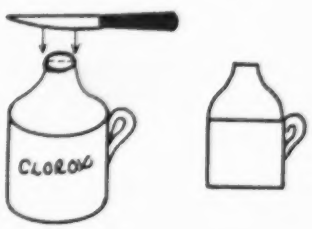
ON YOUR OWN

1. For each of the following sketch the shape of the cut that will result. Draw your sketch beside the figure.

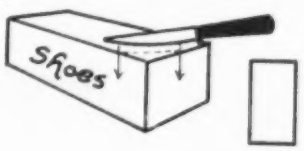
a.



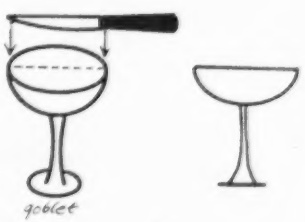
b.



c.

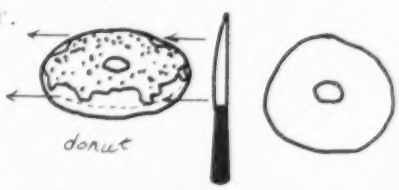


d.

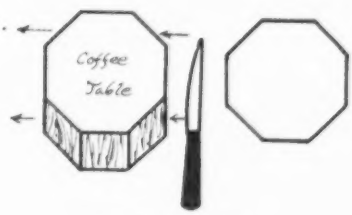


e.

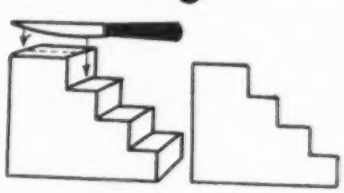
f.



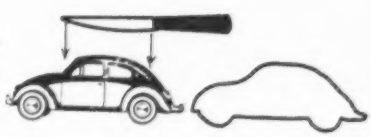
g.



h.



i.



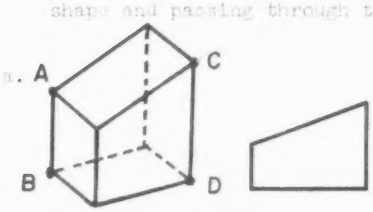
j.

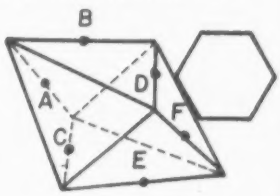
CONTENT AND APPROACH

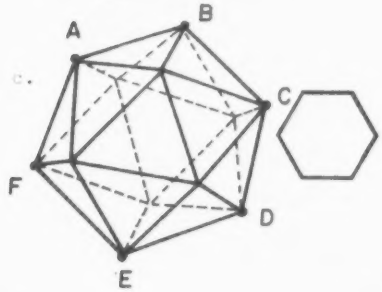
Your students will probably find it difficult to identify the shapes called for in exercise 2. The straw models they constructed in Lesson 1 will be good visual aids.

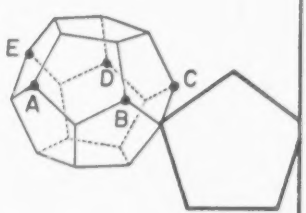
The intention of exercises 3 and 4 is to make the students realize that it is not easy to draw pictures of three-dimensional objects on a two-dimensional surface. Lessons 8 and 9 will present some techniques for drawing space figures. It may be interesting to have them do these exercises again after they have completed Lesson 9.

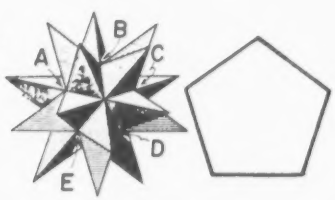
2. For each of the following sketch the shape that would be made by a plane (or a knife) cutting through the shape and passing through the points which are labeled.

a. 

b. 

c. 

d. 

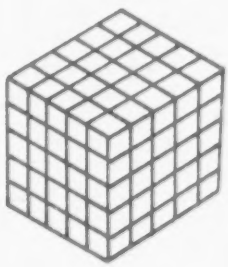
e. 

3. Draw a picture to show how a plane would have to pass through a cube to cut out a triangular shape.

4. Draw a picture to show how a plane would have to pass through a cone to cut an ellipse.

✓ POINT

1. The following picture is made from sugar cubes:



How many sugar cubes were used to build the large cube? 125

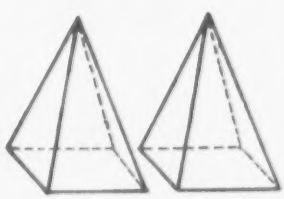
If the large cube is painted red and then taken apart, how many sugar cubes will have:

Four red faces? 0 Three red faces? 8

two red faces? 36 no red faces? 27

2. Every time a plane passes through a sphere its cut is a CIRCLE

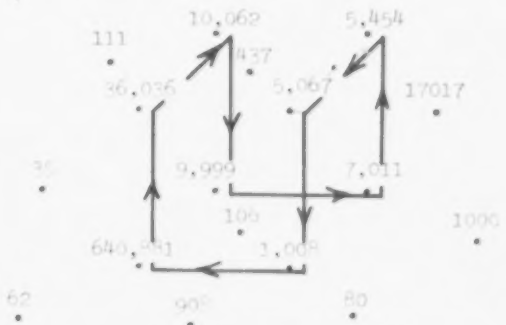
3. The picture shows two square pyramids which are the same size and shape. Imagine one plane passing through both of them at the same time. The plane will cut through parallel to the bases of the pyramids. Which of the following statements are true? c,d



a. The two cuts made will be different in shape.
b. The two cuts will both be triangular in shape.
c. The two cuts will be the same shape and size.
d. The two cuts will have the same shape as the bottom faces of the two pyramids.

TAKING A NUMBER BREAK

1. Which of the following numbers are divisible by 9? (Divisible means the number can be divided by 9 with no remainder.) Circle the dots under those numbers you select.



2. For those numbers which are divisible by 9, find the sums of their digits. (Ex. 5,067→5 + 0 + 6 + 7 = 18). Which of the following is true for all the sums you found? **c**

a. They are even
b. They are odd
c. They are multiples of 9
d. They are multiples of 7

3. Find the sums of the digits for at least two of the numbers which are not divisible by 9. Do these sums have the property you selected in question 2? **NO**

4. Complete the following: "A number is divisible by 9 if the sum of its digits is **is a multiple of 9 or is divisible by 9**."

5. Which of the following are divisible by 9? **a,b,e,h,i**

a. 1,278	d. 557,346	g. 312,020,813
b. 13,770	e. 7,080,615	h. 672,837,888
c. 93,907	f. 43,881,274	i. 5,708,915,127

6. Connect the dots you have circled with line segments. Start at the largest number. Connect the numbers in decreasing order. Then return to the starting point. What Platonic Solid would appear if you added four segments to the figure? **cube** Can you tell which face of the object is closest to you? Look again.

NO

CONTENT AND APPROACH

An excursion into the world of solid geometry would not be complete if the students were not given an opportunity to draw pictures of some of the space figures that they have been studying. Lessons 8 and 9 present the students with this opportunity. It is hoped that these lessons will help the students develop their geometric intuition so that they may more accurately visualize spatial relationships. Also, being able to represent a three-dimensional object on a flat piece of paper usually fascinates students.

This lesson presents some "trick" techniques for drawing simple space figures. You will probably encounter these tricks as the ones you use to represent space figures on the chalkboard in a geometry class.

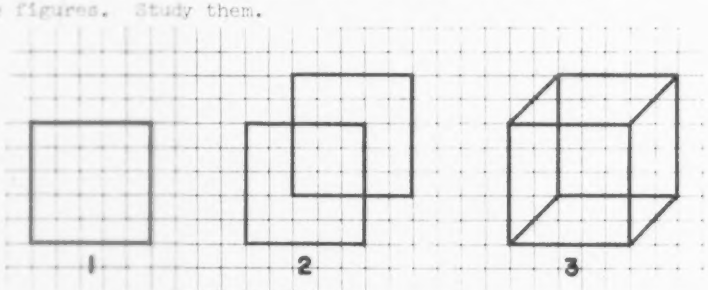
The students should study the examples and then do the [illegible] exercises.

DEPTH IN 2D.

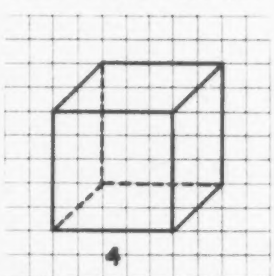
As you have probably found out, it is not easy to draw three-dimensional figures on a sheet of paper (which has only two dimensions). Artists, architects, draftsmen and engineers are often faced with the problem of drawing a three-dimensional scene, building or object on canvas or a sheet of drawing paper. The two-dimensional drawings must give the illusion of three dimensions. There are a number of techniques used in this type of drawing.

Here are some tricks which can be used to draw simple space figures. Study them.

Cube



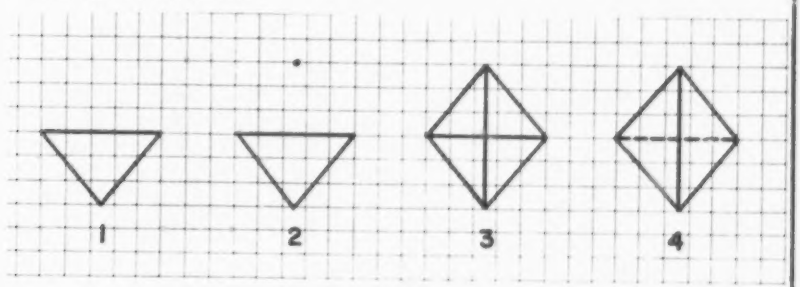
If the cube is solid the back edges would not be visible to the human eye. The lines which are not visible in this view are dotted.



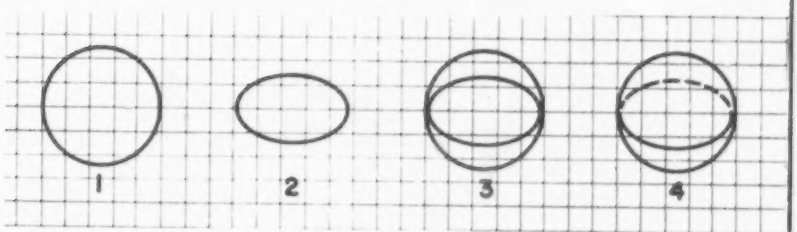
OBJECTIVES

EQUIPMENT AND TEACHING AIDS

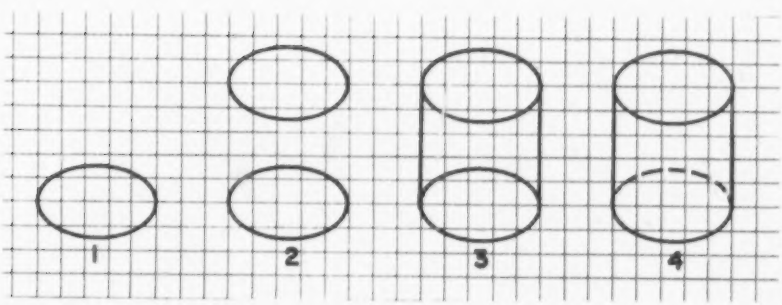
Triangular Pyramid



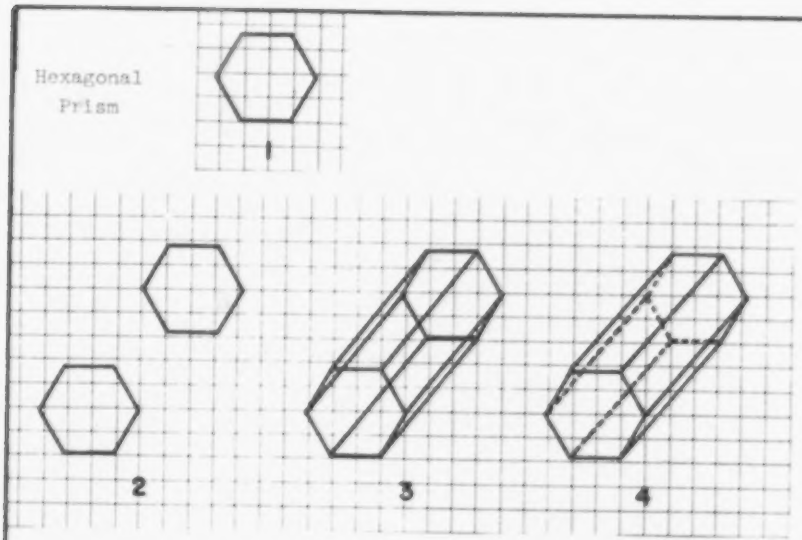
Sphere



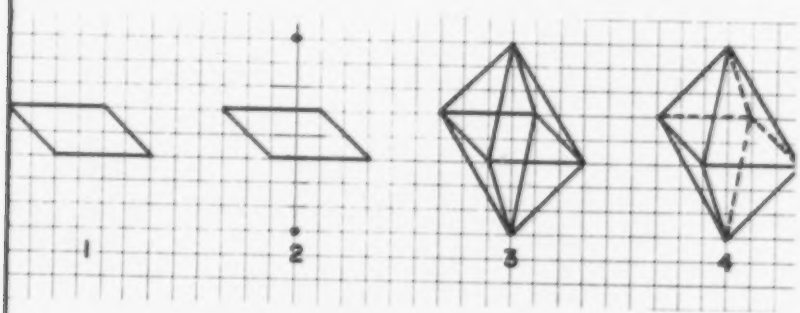
Cylinder



Hexagonal
Prism



Octahedron



CONTENT AND APPROACH

The students should use the engineer's paper included at the back of their booklets for these exercises. They should draw the figures on the blank side and use the grid lines as guidelines. This prevents the dark grid lines from getting in the way of their drawings.

They will need a straightedge. A template which includes circles and ellipses would also be helpful.

Each student should have five sheets of the paper. However, suggest that they use no more than three for these exercises. They will need some later on for Lesson 9.

Number 3 is labelled optional. You may also choose to make number 2 optional.

When you check their drawings look for:

(a) neatness (as much as can be expected);

(b) correct use of dotted lines to indicate lines hidden from view.

ON YOUR OWN

The inserts at the back of the book are for your drawings. They are sheets of engineer's drawing paper. Draw your figures on the blank side; use the tiny squares on the other side as guidelines. You will need a straightedge.

1. Use the trick techniques illustrated to draw:
   a. two cubes of different size
   b. a triangular pyramid
   c. a sphere
   d. a cylinder
   e. a cone
   f. a triangular prism
   g. an octahedron
   h. a shoe box

2. In number 1 you were asked to draw single objects. Use the same techniques to draw:
   a. two cubes (same size) which share an edge
   b. a double-dip ice cream cone
   c. a cube with a circular tunnel drilled through the center
   d. two tetrahedrons which share a face
   e. the Geopolis Micro-Film Library (see page 2)

3. (Optional) Draw a collage which includes at least five different three-dimensional objects.

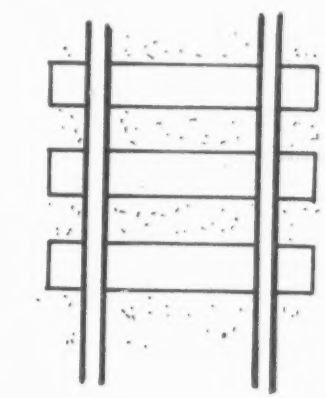
CONTENT AND APPROACH

The "trick" techniques for drawing space figures which were described in Lesson 8 are rigid and require little creativity. Artists use a sophisticated, more creative form of art called perspective drawing.

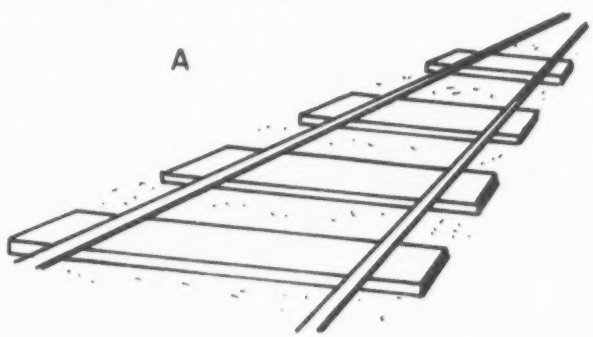
Perspective deals with the phenomenon of appearance. It is used to depict objects as seen from a definite point of view. The artist selects the point of view and draws the picture accordingly.

Once the basic rules of perspective drawing have been learned, the students will be able to depict space figures to better express their ideas. Lesson 9 deals with some of these basic rules. It treats three aspects of perspective drawing: one-point, two-point, and "depth illusion".

PUTTING SOLIDS IN PERSPECTIVE



A



B

A class was asked to draw a picture of railroad tracks. The pictures above were done by two different students.

LESSON 9

OBJECTIVES

1. The student will realize that in a perspective drawing the edges of an object which are actually parallel are drawn to meet at a point (vanishing point) on the line which is at eye-level (horizon line).

2. The student will see that: (a) selection of vanishing points on the horizon line depends on the position of the viewer. (b) placement of an object above or below the horizon line depends on whether the viewer is looking at the bottom or top of the object.

3. Given a horizon line, a vanishing point on that line, and one face of a cube, the student will be able to complete the drawing of the cube.

4. Given a horizon line, two vanishing points on that line and one edge of a cube, the student will be able to draw a recognizable picture of a cube.

5. The student will be able to space vertical lines (telephone poles) or horizontal lines (train tracks) in a perspective drawing to portray an illusion of depth.

EQUIPMENT AND TEACHING AIDS

1. Straightedge (one for each).

*2. Engineer's paper

CONTENT AND APPROACH

This class discussion should generate the following ideas: (1) when an object is viewed by the human eye it may appear to be unlike what it is in reality; (2) the same object will look different if it is viewed from different positions.

Picture A more accurately depicts what railroad tracks are really like. However, they seldom appear this way when viewed by the human eye. One would have to be in a helicopter flying directly over railroad tracks and looking straight down at them.

If we were shown picture A with no other information it would be difficult to identify the object being pictured.

Picture B would more easily be identified as railroad tracks for the eye is more accustomed to seeing tracks from this (or a similar) point of view. The drawing in B is not true to reality. The steel rails seem to converge to a point and the ties are not all the same size. Yet, it looks more like railroad tracks than figure A does. It is not really possible to determine which picture is more "correctly" drawn for we do not know if the class was given any specific instructions about drawing their pictures.

Question 4 may cause some confusion. Picture B was the intended answer. However, to see the tracks as drawn in figure B you would not be standing directly on the tracks but rather off to the right of the tracks. This may prove to be a profitable mistake. The resulting discussion should bring out the fact that pictures can be drawn to portray different points of view. Suggest they compare picture B with the picture on page 68. The development in Lesson 9 should help the students understand how these different effects are achieved.

THINK ABOUT IT

1. Railroad ties are actually the same size. Which picture portrays this notion better?

2. The steel rails of railroad tracks are the same distance apart at all points. Which picture portrays this better?

3. Which picture do you think is more correctly drawn?

4. If you were standing on the tracks and looking off into the distance, which picture best portrays what you would see?

5. Where would you have to be to see the railroad ties as they are drawn in Figure A?

The student who drew picture B used a technique called perspective. This technique is used by artists to achieve a feeling of space, of depth and of the third dimension within the limits of a flat drawing surface.

CONTENT AND APPROACH

Basically, in the one-point technique all lines of an object converge and disappear at a point on the horizon (eye-level line). This technique usually produces a more distorted image than the two-point technique. This is particularly true if the vanishing point is moved too far from the center of the picture.

One-point perspective is commonly used in drawings of train tracks or a series of telephone poles. It is also used to depict the interior of a room.

THINGS TO DISCUSS

What are some other examples of the use of one-point perspective? Ask the students to bring in some photographs which portray one-point perspective.

ONE-POINT PERSPECTIVE

Steel rails of a train track are always the same distance apart. If you were to stand on the tracks (we don't suggest you try this) and look off into the distance, the rails would seem to come together at a point.

This illusion is an example of one-point perspective. Parallel lines which extend away from the observer seem to meet at a point (vanishing point) on a line which is at eye-level (horizon line).

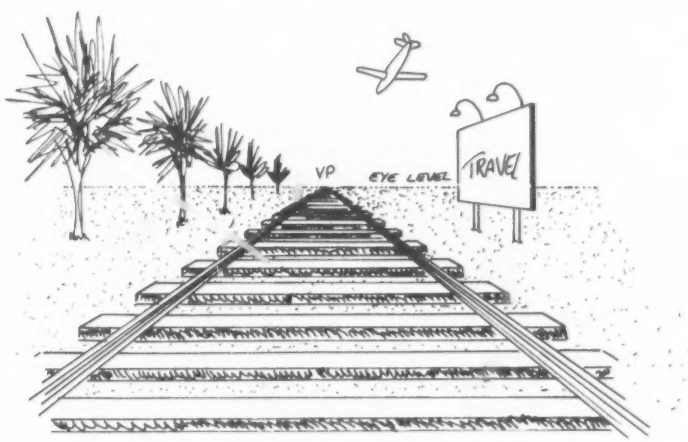


figure 1

This one-point perspective technique is frequently used by interior designers who want to describe their plans for decorating a room.

CONTENT AND APPROACH

The statements printed at the top of page 70 are the two most important conclusions which should be reached as a result of discussing these Think About It questions.

ANSWERS

1. The vanishing point is located at the center of the picture. The viewer is standing on the tracks and looking off into the distance.
2. The viewer is looking at the tracks from the top. The tracks are located below the horizon line.
3. You see the bottom of the airplane. It is located above the horizon line.
4. In figure 2 the vanishing point is located to the left of the center. The viewer is standing to the left of the center of the picture.

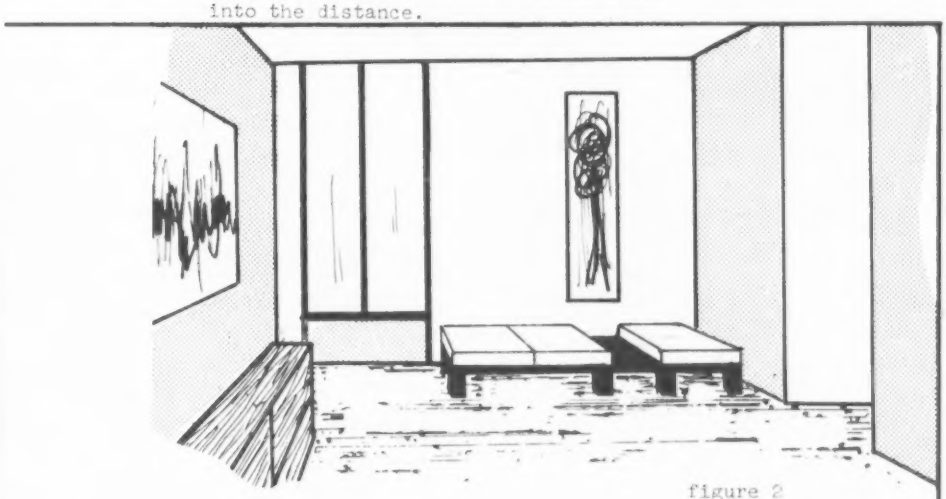


figure 2

Notice how the lines of the various objects (including the room) which in reality are parallel, converge to a point (vanishing point). This point is located along the eye-level line.

THINK ABOUT IT

1. In figure 1 where is the vanishing point located with respect to the center of the picture? Where is the viewer standing?
2. Are you looking at the railroad tracks from the top or the bottom? Where are the tracks located with respect to the horizon line (above or below)?
3. Do you see the top or the bottom of the airplane? Where is the airplane located with respect to the horizon line?
4. In figure 2, is the vanishing point at the center of the picture? Where is the viewer standing?

CONTENT AND APPROACH

The students should read through the step-by-step instructions which begin on this page and end on page 72. Suggest they use a sheet of engineer's paper to duplicate the steps as they read through them.

Something To Keep In Mind:

The selection of a vanishing point on the horizon line will depend on the position of the viewer.

Placing an object above or below the horizon line will depend on whether the viewer is seeing the top or bottom of the object.

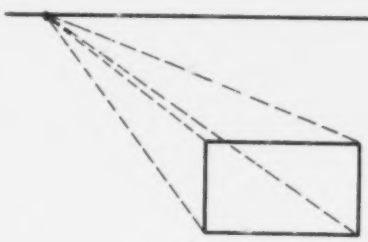
Study the following steps for drawing a box using the one-point perspective technique.

1. Draw your horizon line.

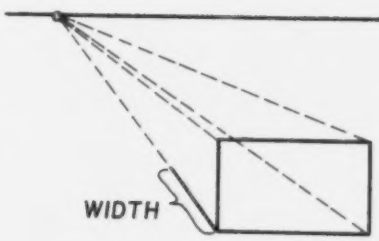
2. Select a vanishing point.

3. Draw the front face of the box.

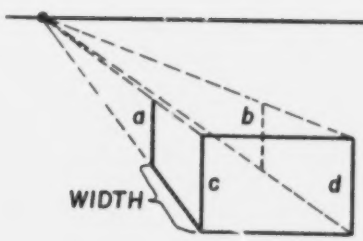
4. From each corner of the front face draw a dotted line to the vanishing point.



5. Select a width which is pleasing to your eye.



6. Draw in edges a and b parallel to front edges c and d.



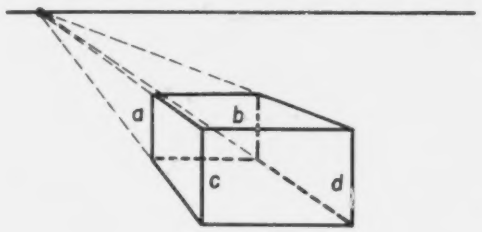
CONTENT AND APPROACH

The step-by-step instructions illustrate how to draw a box from one point of view. In the On Your Own exercises the students are asked to draw three different views of a cube. Show the students transparency G_{17} before they begin the exercises. This transparency illustrates how the view of a box changes as the position of the front face is moved with respect to the horizon line. You might also draw a picture of a box (on the chalkboard) which will show them how the view is altered when the vanishing point is moved to a different location on the horizon line.

All of the students should do exercise 1. Exercises 2 through 5 can be considered optional. Use your own discretion in assigning these.

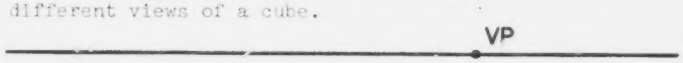
You will have to use your own judgment in correcting these exercises.

7. Complete the box by drawing in the other edges. Remember — those lines which are hidden should be dotted.

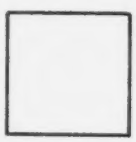


ON YOUR OWN

1. Use the one-point perspective technique to draw three different views of a cube.

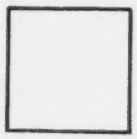


A.



B.

C.



(For exercises 2, 3, 4 and 5 use the engineer's paper which is included at the back of this booklet.)

2. Draw a picture of a pair of dice using the one-point technique.

3. Draw a picture which gives the illusion of looking down a hallway or tunnel. Locate the vanishing point at the center of the picture.

4. Sketch a picture of your school or some other building using the one-point technique. Place the vanishing point off center — to the right.

5. Sketch a picture of a highway, country road or a busy city street using one-point perspective.

CONTENT AND APPROACH

The face of an object drawn using the one-point technique is parallel to the horizon line. The two-point technique enables the artists to portray objects which are oblique to the horizon line. When using the one-point method one starts with the face of an object; with the two-point, one starts with an edge of the object. From the endpoints of this edge, lines converge to the right and left toward 2 points on the horizon line.

The two-point techniques is more frequently used. Some examples of its use are pictures of: an aerial view of a city; a corner of a building; a machine part.

Let the students read through the step-by-step development on pages 75 and 76. They should carry out the steps as they read them on a sheet of engineer's drawing paper.

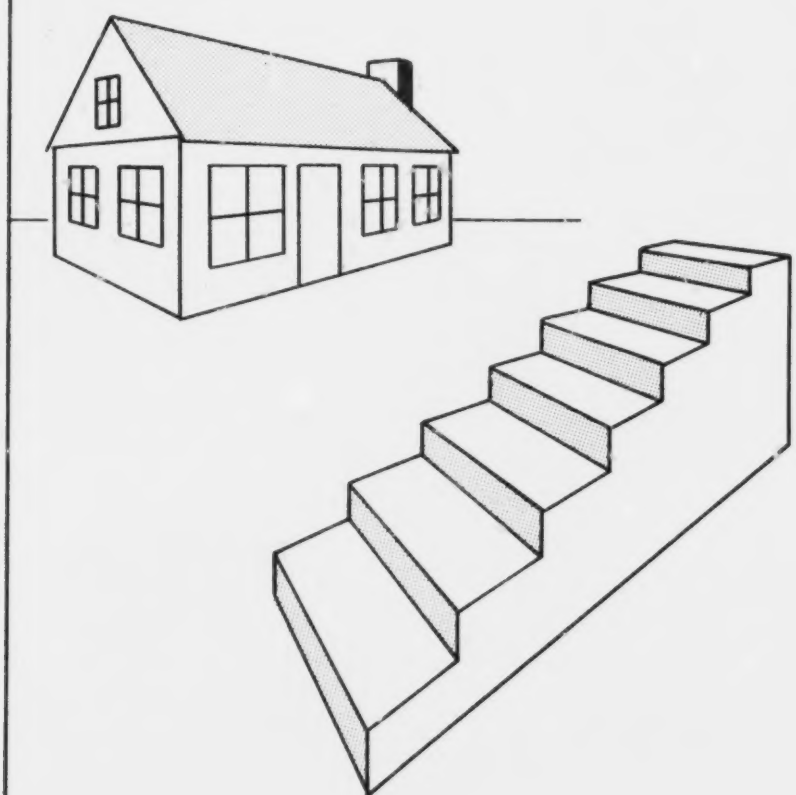
THINGS TO DISCUSS

What are some other examples of the use of two-point perspective? Ask the students to find photographs which depict two-point perspective.

TWO-POINT PERSPECTIVE

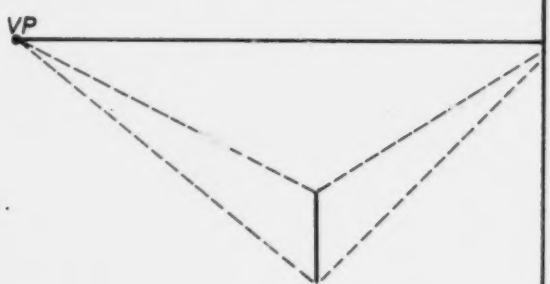
Artists and architects more frequently use a two-point perspective technique in their drawings of three-dimensional objects. In this form of perspective drawing, the parallel lines of an object converge to two different points along the horizon (eye-level line).

The following are examples of the use of the two-point perspective technique.



Study the following steps for drawing a box using two-point perspective.

1. Draw your horizon line.

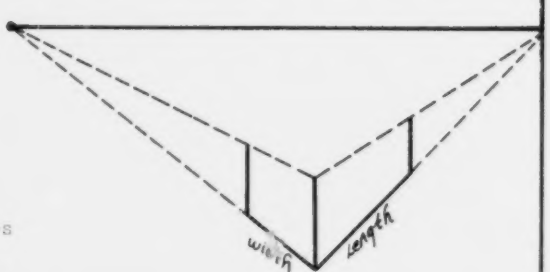


2. Select two vanishing points.

3. Draw an edge of the box.

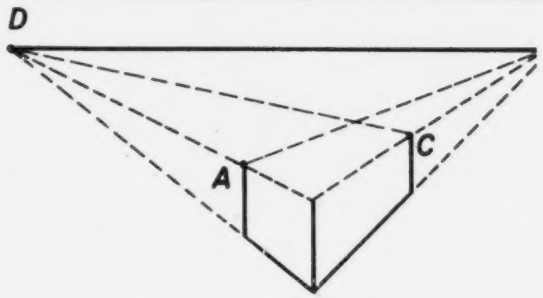
4. Connect the endpoints of the edge to the vanishing points.

5. Select a length and width for the box.

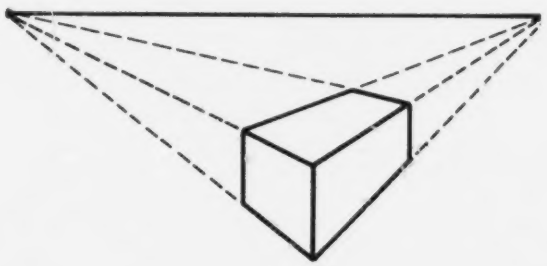


6. Draw in the edges parallel to the front edge.

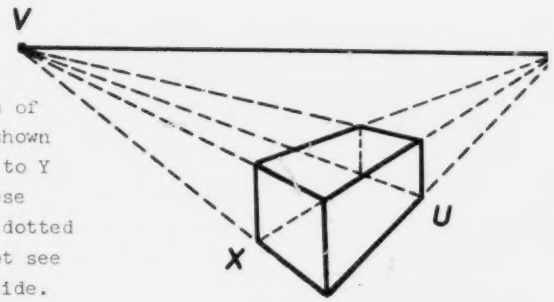
7. Draw a dotted line from A to B and from C to D.



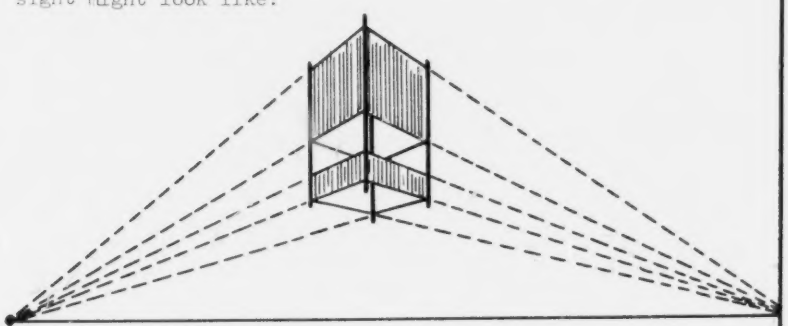
8. Draw in the edges of the box which you can see.



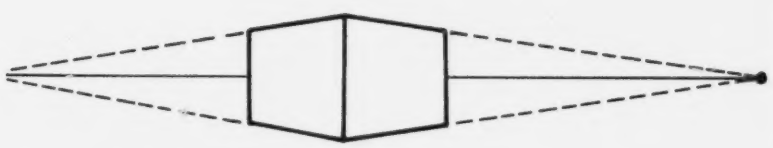
9. The hidden edges of the box can be shown by connecting X to Y and U to V. These edges should be dotted for you could not see them from this side.



A picture of a box kite in the air above the line of sight might look like:



If you are drawing a picture of a box which hides part of the horizon it might look like:



Again...

1. The location of the vanishing points depends on the position of the viewer.
2. The placement of an object above, below or at the horizon line depends upon whether you want to show the top, bottom or just the sides of an object.

CONTENT AND APPROACH

All of the students should do exercises 1 through 3. Use your own judgment in correcting their work.

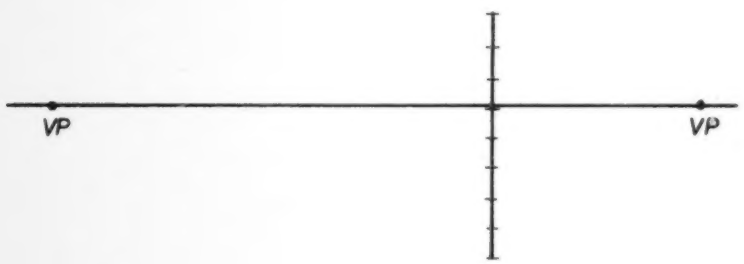
Exercises 4 and 5 can be considered optional. If they are assigned, the students should try to do them without using engineer's paper. (They will not always have this type of paper available to them.)

ON YOUR OWN

1. Use the two-point perspective technique to draw a picture of a cube.

2. Use the following to draw a picture of a shoe box.

3. Use the following to draw a picture of a eight story rectangular-shaped building.



4. Draw pictures of the seven Soma Cube puzzle pieces that you built in Lesson 6.

5. Draw a picture of a staircase using two-point perspective.

CONTENT AND APPROACH

The students have had some experience with making objects look three-dimensional even though they are drawn on a two-dimensional surface. Now they will be given some instruction for spacing lines to make objects look like they are way off in the distance. This spacing technique is often combined with the one-point and two-point techniques to draw pictures which are realistic to the eye.

THINGS TO DISCUSS

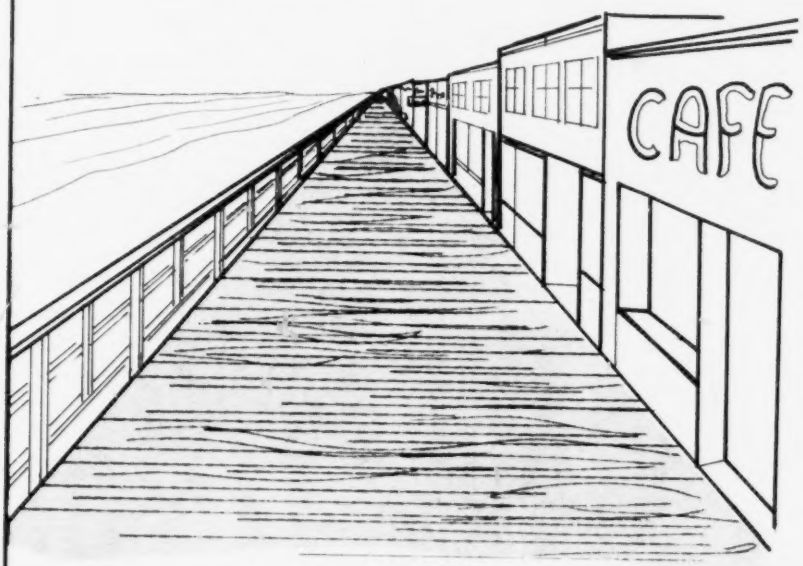
Do you think there is a system for spacing lines to give the illusion of distance or does the artist simply place them where his eye perceives they should be?

If most of your class feel that an artist does not need a system, ask them to draw a picture of train tracks before they read the next two pages.

DISTANCE ILLUSION

Another aspect of perspective drawing is to picture objects so they appear to be at different distances from the viewer. This is accomplished by making objects smaller as they recede from the viewer.

For example, an artist sitting at the entrance to th boardwalk in Atlantic City might sketch the scene like this:



The artist knows that the wooden planks are all the same size to give the illusion of distance. The same is true of the side rail posts.

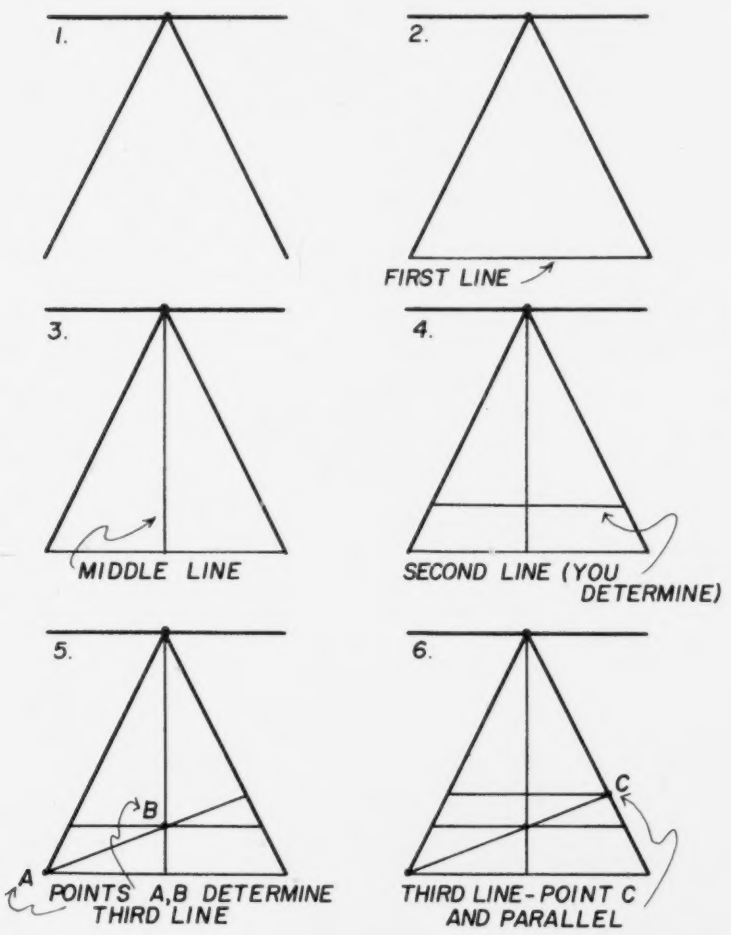
CONTENT AND APPROACH

If your students are given an opportunity to draw a picture of train tracks without using a system for spacing the lines, they will find that it is not easy to do. Their pictures will probably look distorted.

Pages 91 and 92 illustrate a method which can be used to space lines in a drawing to give an illusion of depth (or distance from the viewer.) After they have had a chance to study the eight steps you might briefly discuss the method used before they begin the On Your Own exercises.

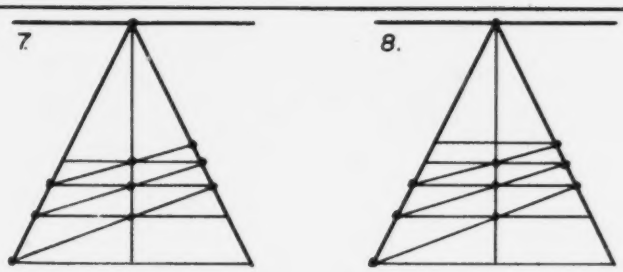
In this booklet we only apply a spacing technique to pictures drawn using one-point perspective. Your students may want to explore similar methods which can be used when they draw pictures using two-point perspective.

The following steps show a method which can be used to achieve this distance effect with objects which are parallel to the horizon line. The trick to using this technique is spacing lines in relation to the vanishing point.

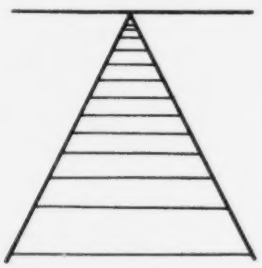


CONTENT AND APPROACH

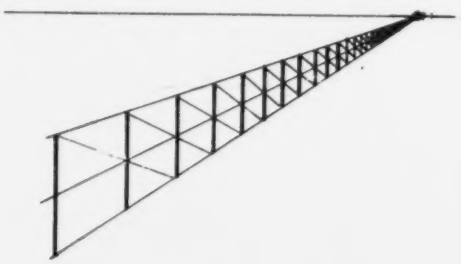
Ask the students to complete the picture in step eight. The lines in their completed picture should be spaced like the series of lines in the illustration which is in the middle of page.



If you continue the process and erase the middle line and the diagonal lines you will get a picture which appears to be train tracks or the planks on a boat dock. (Hint-- do you need to keep drawing the diagonal lines?)



This same process can be used with objects which are situated perpendicular to the horizon line (side rail posts, telephone poles).



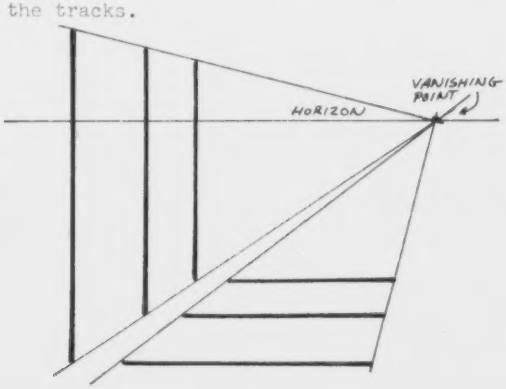
ON YOUR OWN

The students are asked to complete a horizontal and vertical series of lines in exercise 1 and a checkerboard in exercise 2. Suggest that they draw the middle lines and oblique lines very lightly in pencil so that these lines can be erased.

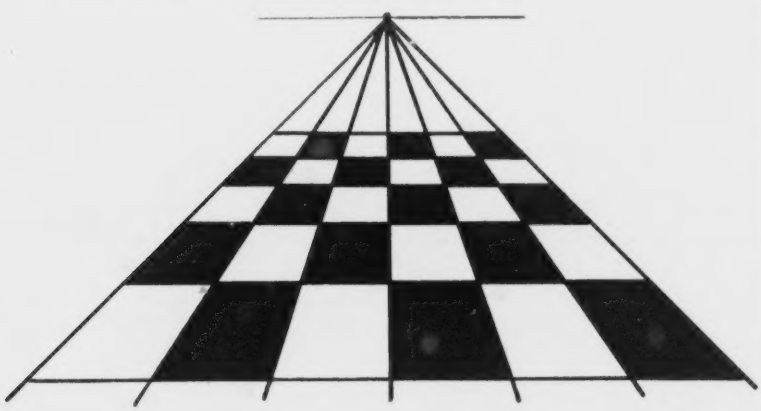
The students should all work exercises 1 and 2. Exercise 3 may be considered optional.

ON YOUR OWN

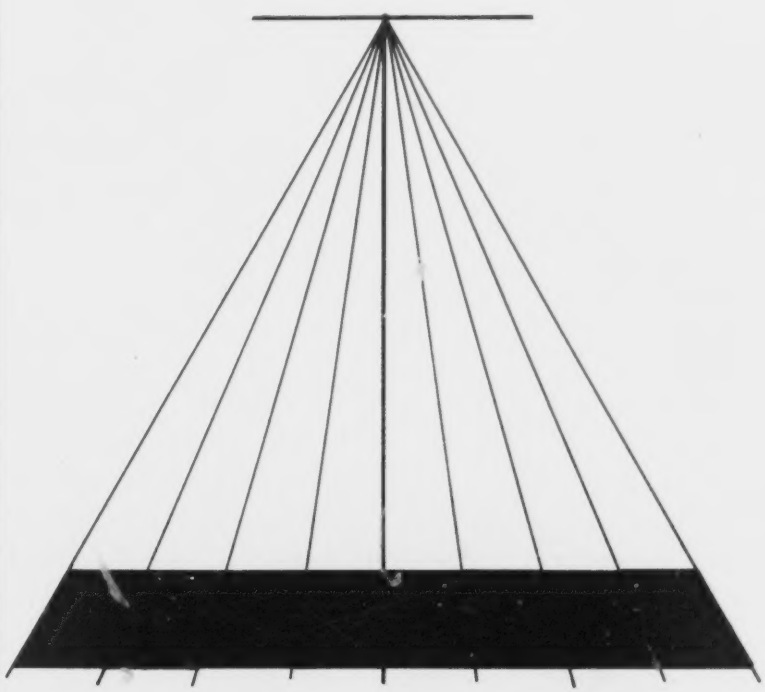
1. Use the perspective technique just described to complete this picture of train tracks with telephone poles along one side of the tracks.



2. A checkerboard, chessboard or tile floor may also be drawn using this perspective technique.



Complete the picture of an 8×8 checkerboard below: (Hint--use the technique described to place your horizontal lines.)

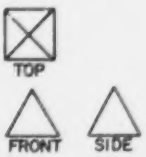


CONTENT AND APPROACH

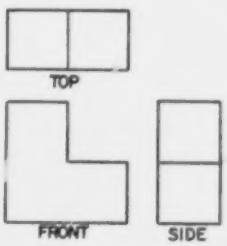
Lessons 8 and 9 have considered two different forms of drawing. The "trick" techniques presented in Lesson 8 can be compared to what is called isometric drawing. Lesson 9 has dealt with some basic ideas of perspective drawing.

You might also make your students aware of a third form of drawing which is known as orthographic. Draftsmen, architects and engineers use orthographic drawing to describe solid three-dimensional objects. They depict an object by drawing three distinct two-dimensional figures which represent the top, front and side view of the object.

For example, they would describe a block of wood shaped like a square pyramid as follows:



An L-shaped block of wood would be characterized by the following.



Note: If you have any drafting students in class who are familiar with orthographic drawing you might ask them to present some other examples.

3. Use the distance illusion technique and either the one-point or two-point technique (or both) to draw the following:

   a. a scene depicting a country road (you might include billboards, telephone poles, picket fences or a line of cars moving down the road away from the viewer).

   b. a scene showing the buildings along a street either in a downtown area or in a residential area.

   c. a scene showing people waiting in line to get tickets for a movie.

CONTENT AND APPROACH

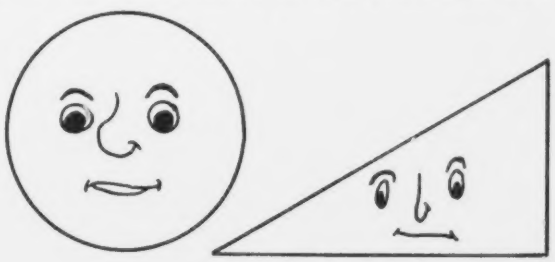
As was pointed out in the Preface of this Teacher's Guide the approach to the study of non-metric geometry in this booklet is different from the conventional treatment of the subject. Instead of starting with the abstract idea of a point and proceeding to the line, to the plane, and then to space, this booklet and the succeeding booklet (Geometric Excursions II) proceed in the opposite direction.

The first nine lessons of this booklet concentrated on the study of space figures. This lesson introduces them to the two-dimensional world. (This world will be explored in depth in Geometric Excursions II.) In the search for an interesting way to make the transition from three dimensions to two dimensions, the idea of shadows came up. The shadow shapes produced by shining a light on the straw models built in lesson 1 conveniently introduces the students to a variety of plane geometric figures.

In the discussion on this page the students are asked to think about living in a two-dimensional world. They might describe their new world as a flat sheet of paper or simply as a flat surface. A good description of the way they would look would be "like their shadows". Life would certainly be different. A person would be very much restricted — no more riding motorcycles, watching TV or playing basketball. The buildings of Geopolis would look like the shadow shapes illustrated on the following pages. Suggested reference book — Flatland: written by Edwin Abbott and published by Dover Publications, New York.

THE 3D / -1D WORLD

Suppose the world we live in suddenly lost a dimension. Without warning, the dimension of breadth would be eliminated. Imagine yourself an inhabitant of this two-dimensional world.



Do you often get that flattened out feeling?

Class Discussion:

1. Describe your new world.
2. What would you look like?
3. How would your life be different in the world of two dimensions?
4. What would the buildings of Geopolis look like in a two-dimensional world?

Let's call our new two-dimensional world Shadowland. Think about standing in front of a lamp and letting the light rays project your image onto the wall (a flat two-dimensional surface). What do you see? If you change your position, what happens? Is your shadow a good picture of the way you would look in a two-dimensional world?

LESSON 10

OBJECTIVES

1. To introduce the students to some of the plane geometric figures which will be studied in Geometric Excursions II.
2. Given one of the straw models (built in lesson one) and a light source the student will be able to move the model into different positions in order to produce different shadow shapes.
3. The student will be able to draw a picture of at least 3 of the shadow shapes that would be produced by shining a light on a straw model of a cube or an octahedron. He will be able to do this without actually shining a light on the model.

EQUIPMENT AND TEACHING AIDS

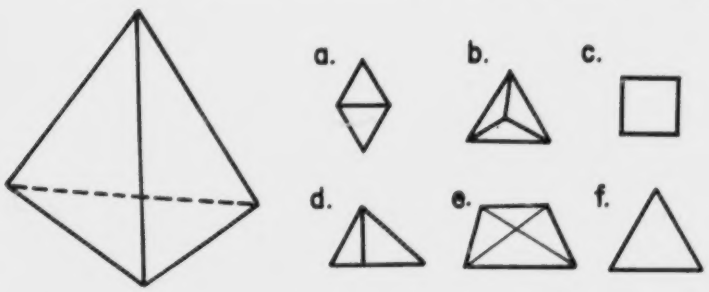
1. Straw models (Pre-lesson 11).
2. Overhead projector and screen.

CONTENT AND APPROACH

These exercises are difficult. It is hard to look at a space figure and try to visualize the shadow shapes it would cast on a wall. The intent of these exercises is to get the students thinking before they actually experiment with shining a light on the straw models.

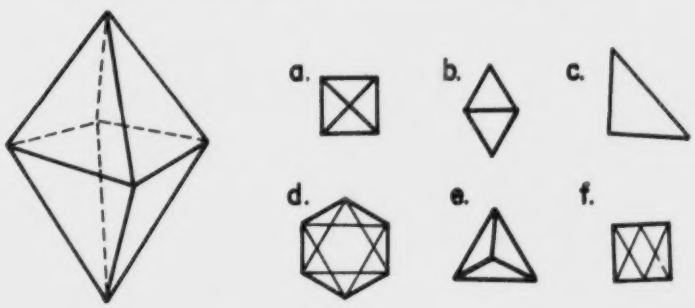
EXERCISES

1. Which of the following shadow shapes do you think can be produced by holding a straw model of a tetrahedron in different positions in front of a light source? **b,d,e,f**



TETRAHEDRON

2. Which of the following shadow shapes do you think can be produced by shining a light on a straw model of an octahedron held in different positions? **a,b,d,f**



OCTAHEDRON

CONTENT AND APPROACH

Let the students do the experimenting in these exercises. Let them hold the models up in front of the lamp of the overhead to project different shapes on the screen. Ask them to name the shapes they see (e.g. square; hexagon with six equilateral triangles in the interior.)

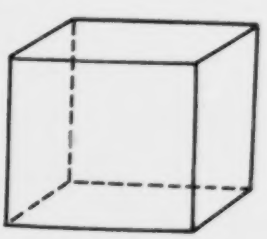
CLASS ACTIVITY

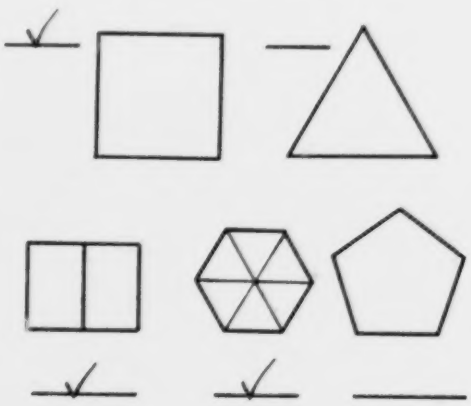
Together with your teacher and your classmates, experiment to see how many different shadow shapes can be produced by shining a light on some of the straw models you constructed in Lesson 1. Use the lamp of the overhead projector as a light source. Below is a checklist which gives a picture of the three-dimensional figure and possible shadows which might be produced by holding the figure in different positions in front of the lamp. Check the shapes which you see on the screen. (First you might check your answers to the preceding exercises by experimenting with the tetrahedron and octahedron.)

Space Figure

Possible Shadows

1.



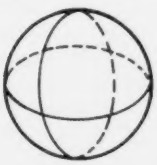


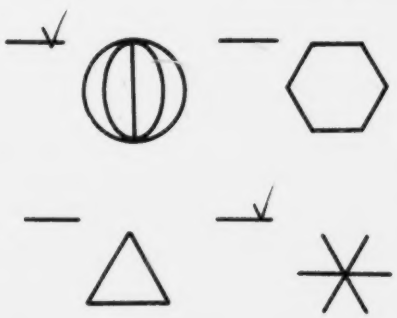
(For each of the space figures, if you see any other shadows which are not illustrated sketch them in any available space.)

Space Figure

Possible Shadows

2.

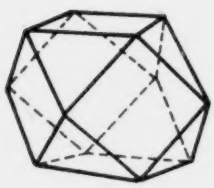


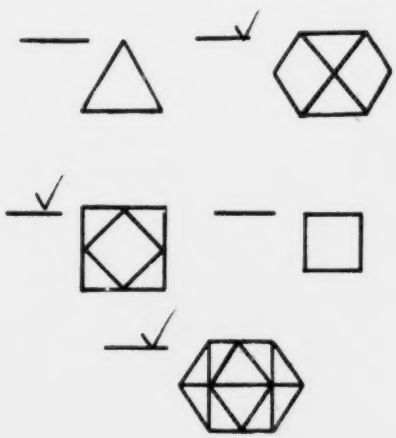


Space Figure

Possible Shadows

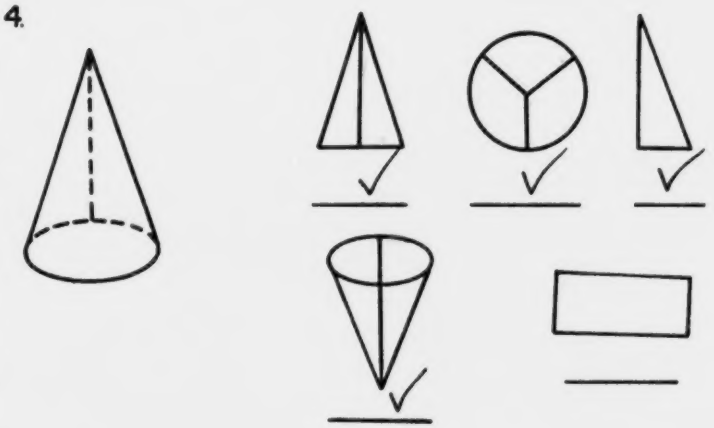
3.





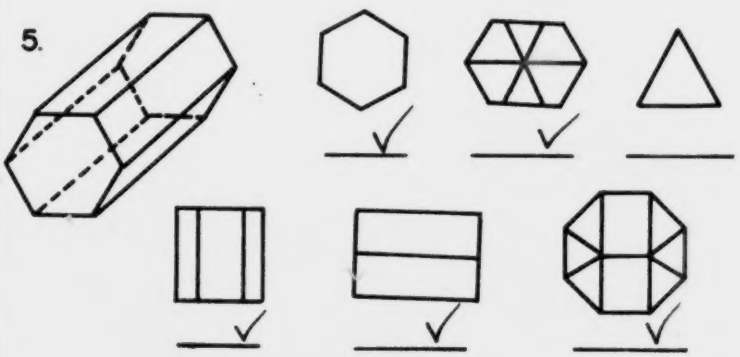
Space Figure Possible Shadows

4.



Space Figure Possible Shadows

5.

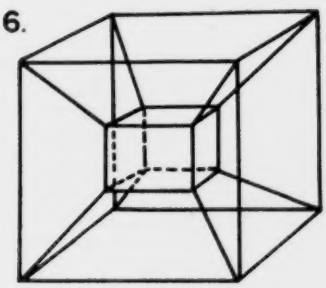


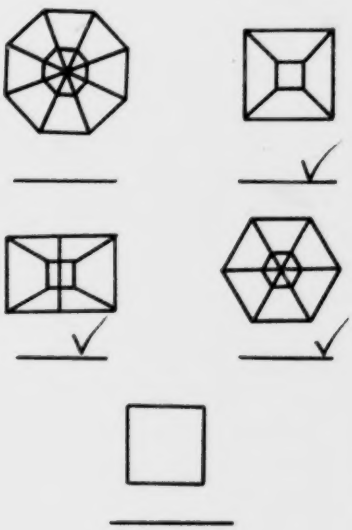
We hope the students have enjoyed this excursion and that you have enjoyed directing them through the various phases of the journey. Glad to have had you aboard!

Space Figure

Possible Shadows

6.



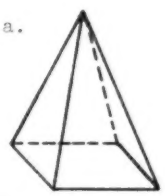


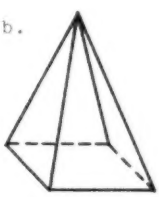
This brings to an end our first excursion into the world of geometry. Our next journey will take us into the world of two dimensions where we will investigate some of the properties of the shadow shapes you have seen in this last lesson. We hope you have enjoyed your excursion. Glad to have you abcard!

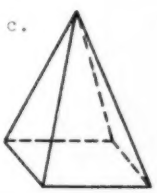
Before you unfasten your seatbelts-

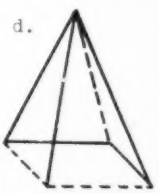
POINT

1. Which of the following sketches of a square pyramid have all the hidden lines drawn correctly? **c**

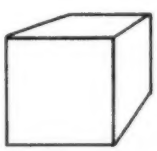
a. 

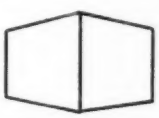
b. 

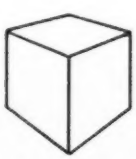
c. 

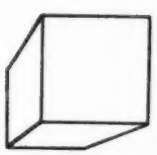
d. 

2. Which of the following cubes were drawn using the one-point perspective technique? **a,d**

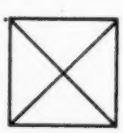
a. 

b. 

c. 

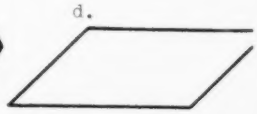
d. 

3. Which of the following shadow shapes could be produced by shining a light on a straw model of a square pyramid held in different positions? **a,b**

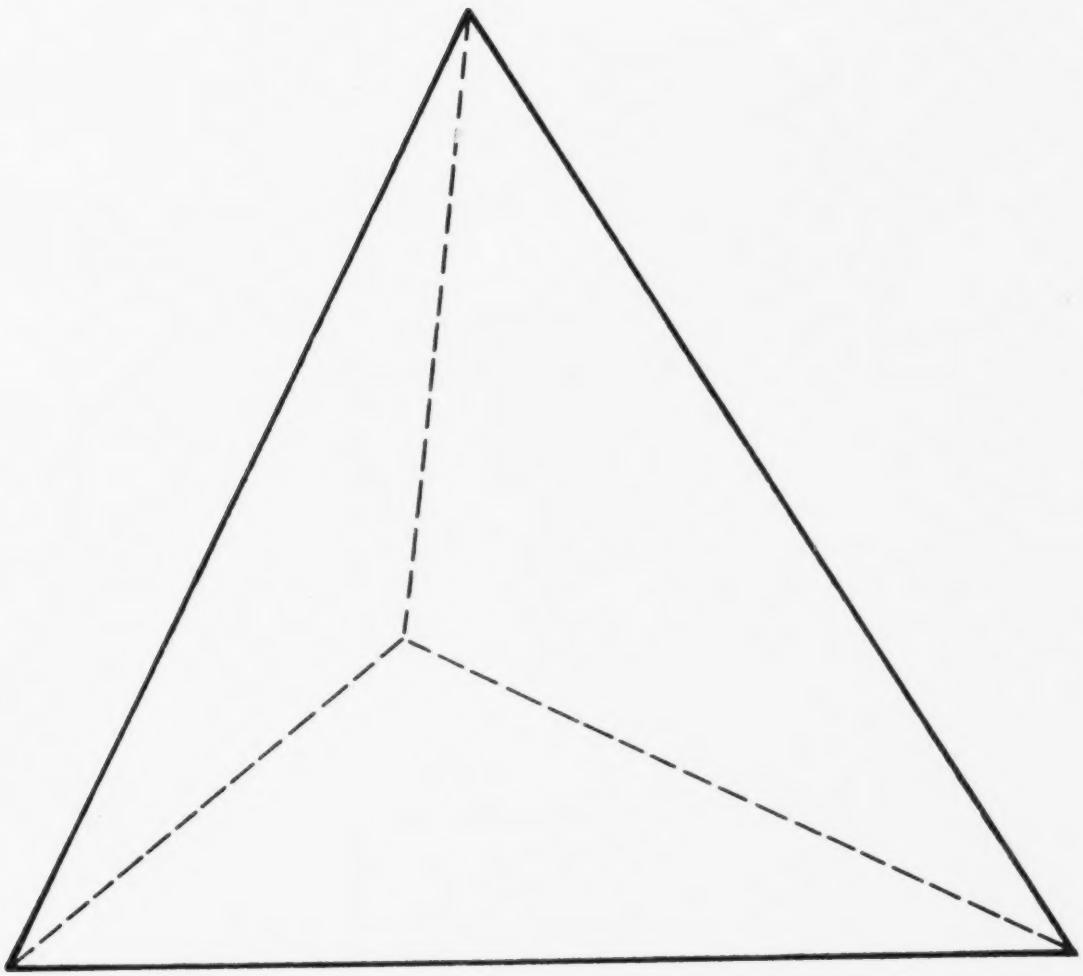
a. 

b.

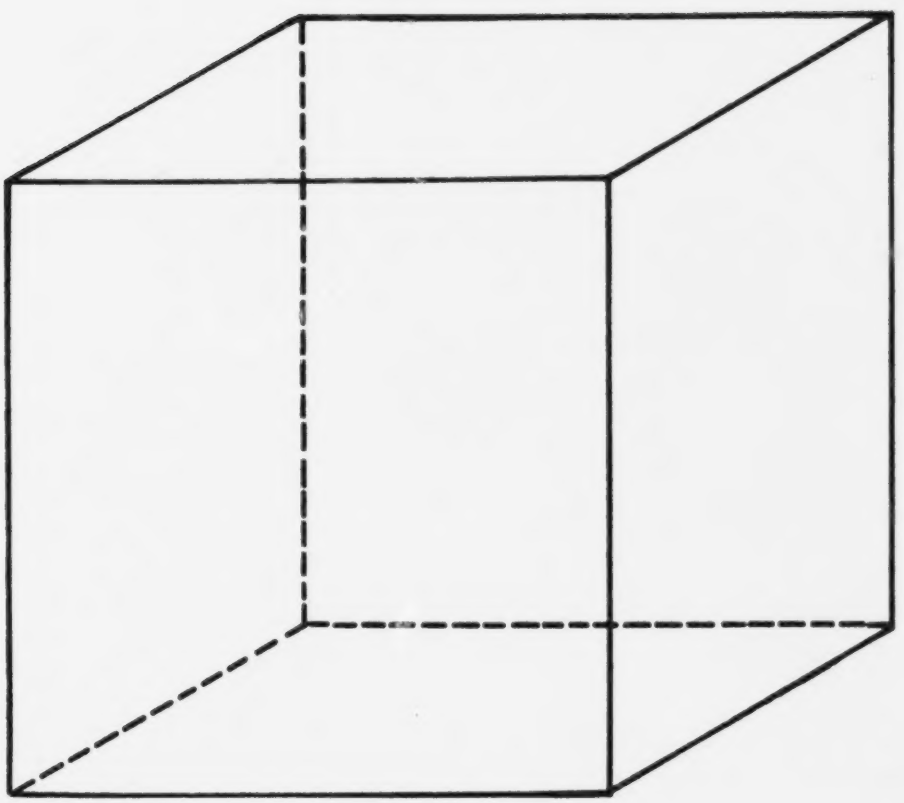
c.

d. 

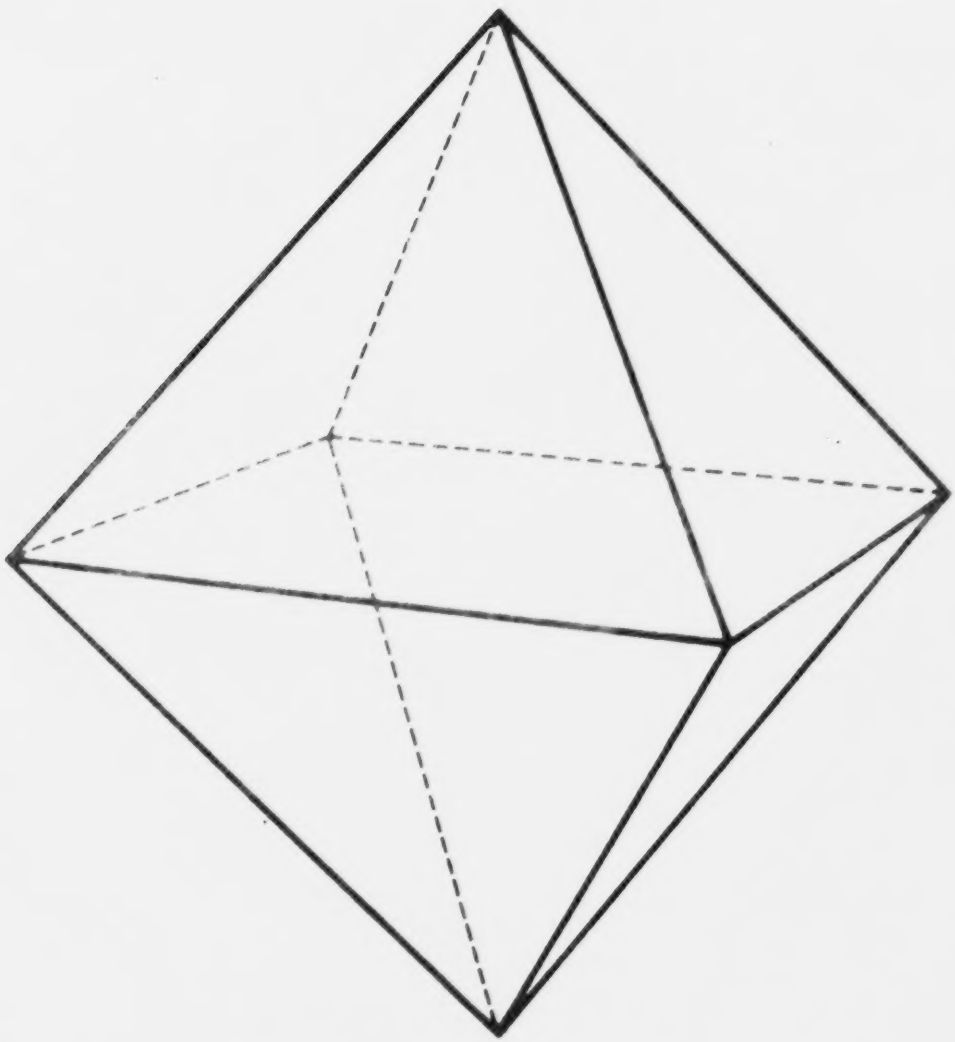
M-i



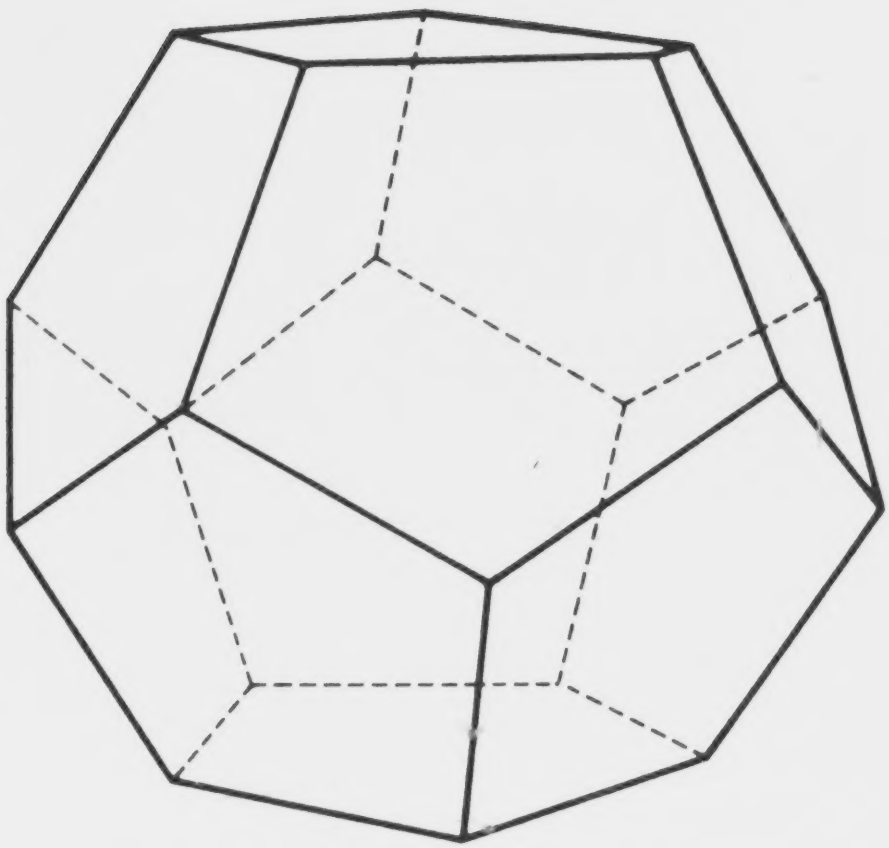
M-2



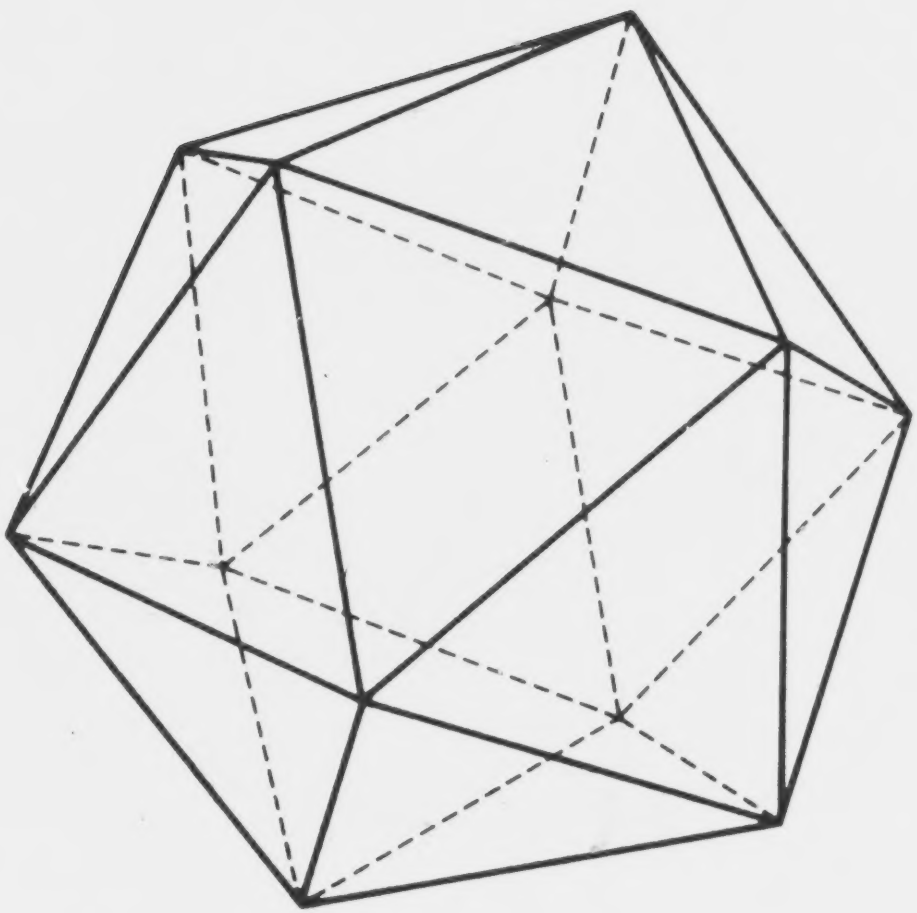
M-3



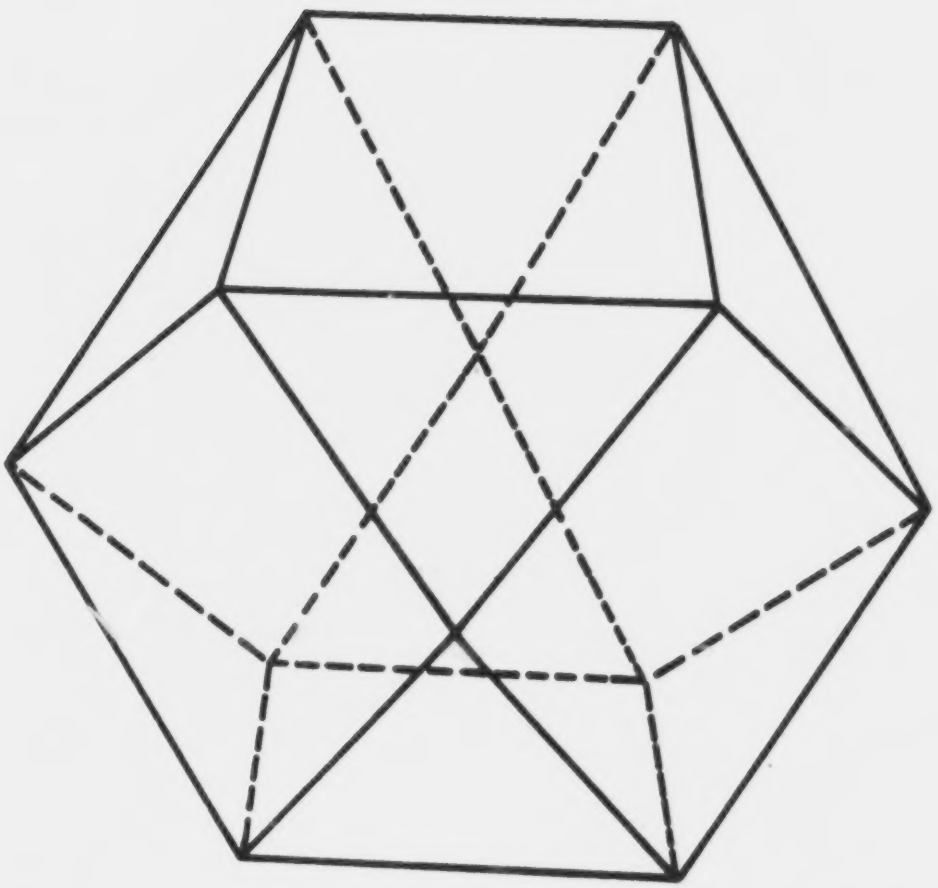
M-4

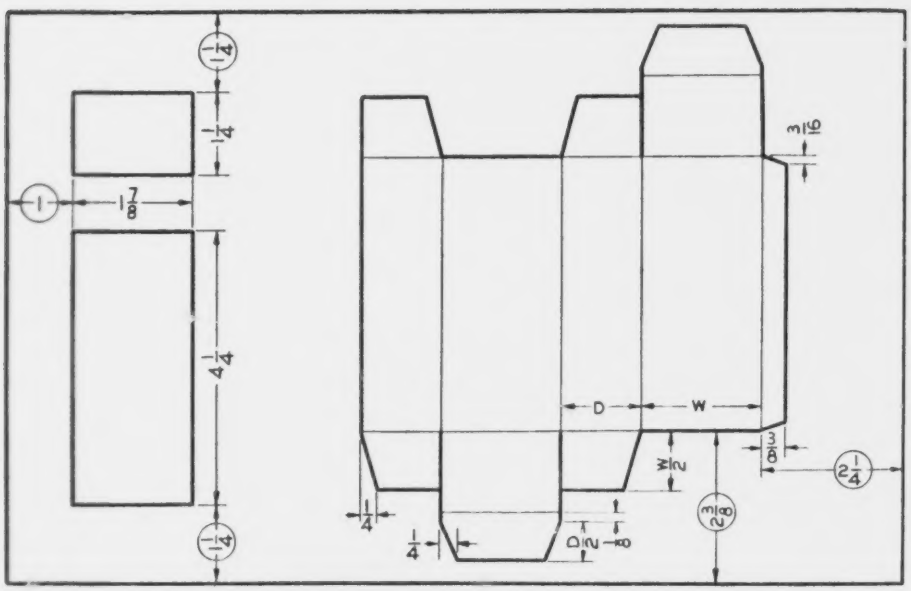


M-5

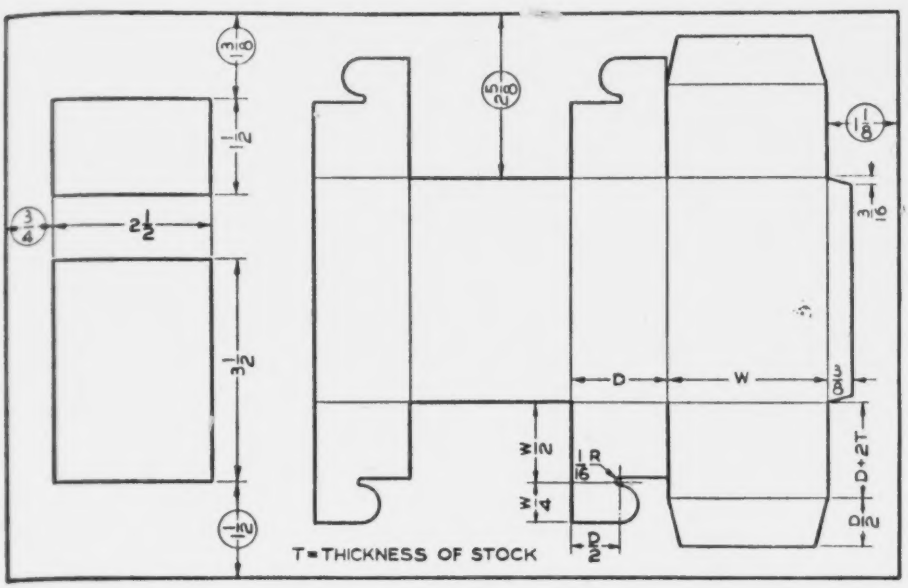


M-6

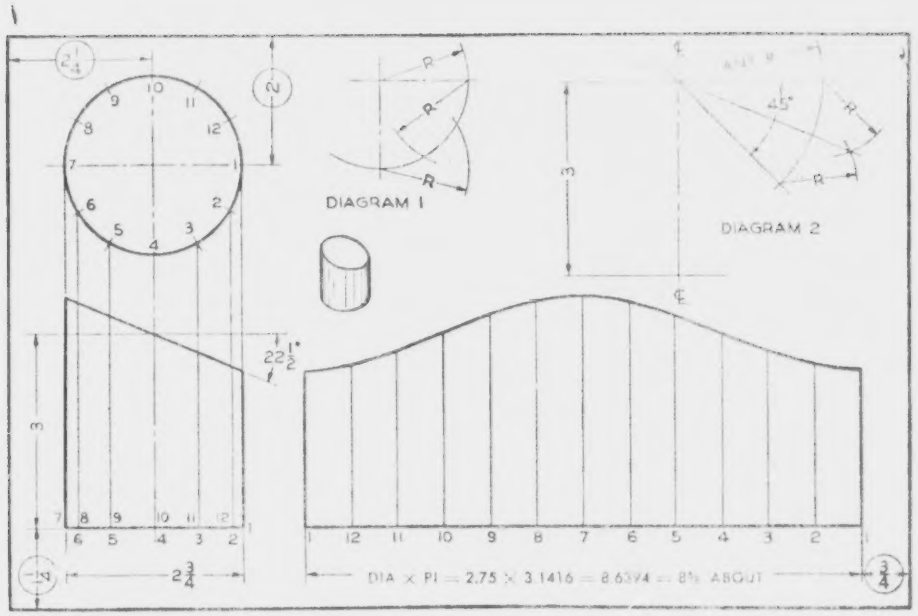




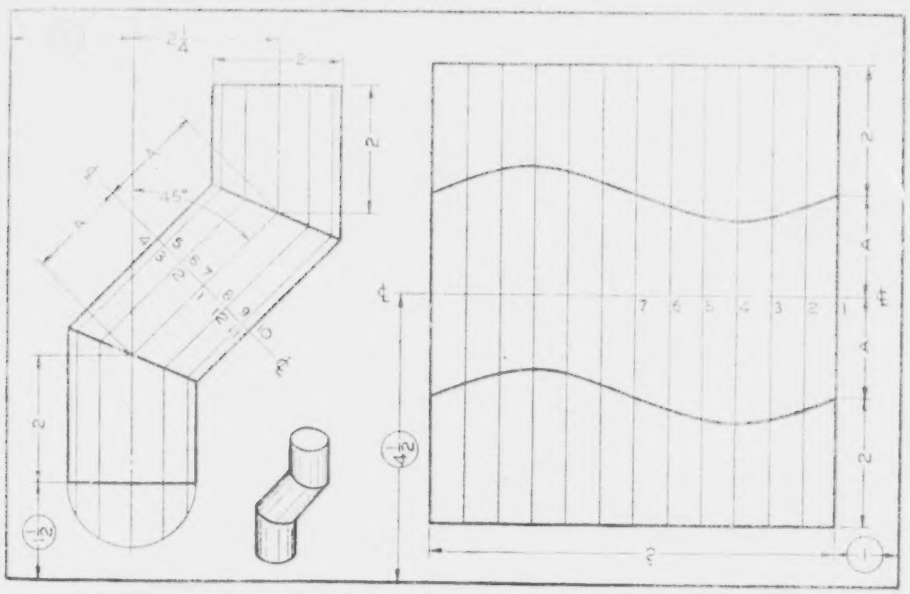
PAPER CARTON



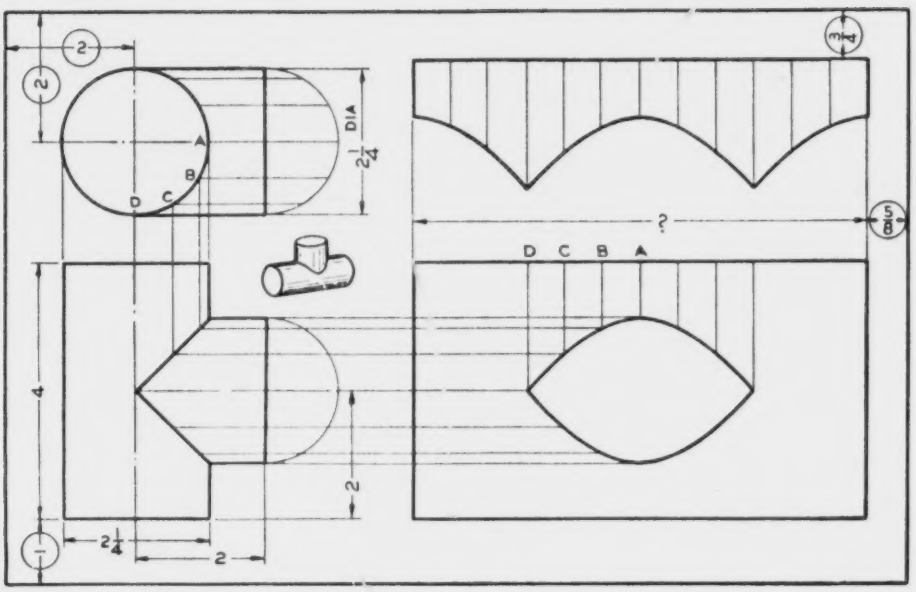
LOCK-FLAP CARTON



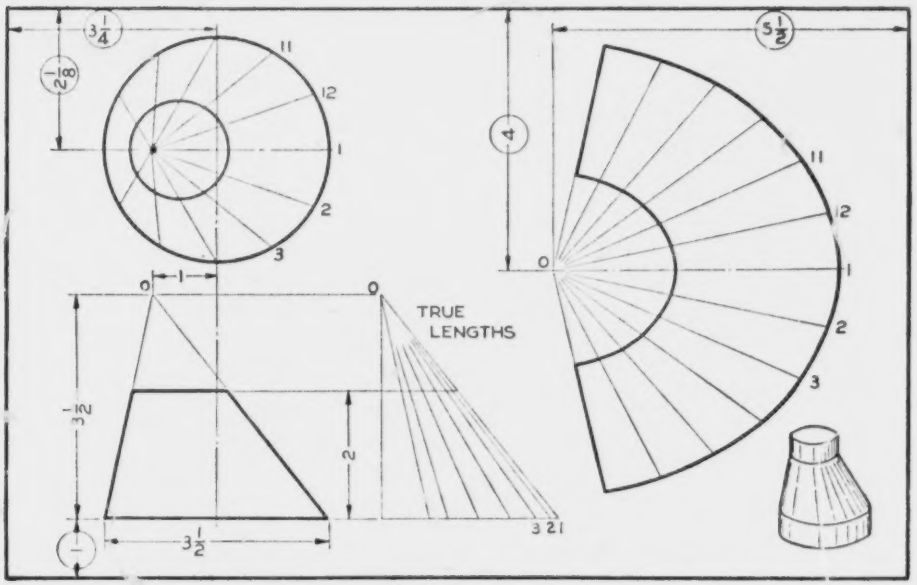
TRUNCATED CONE



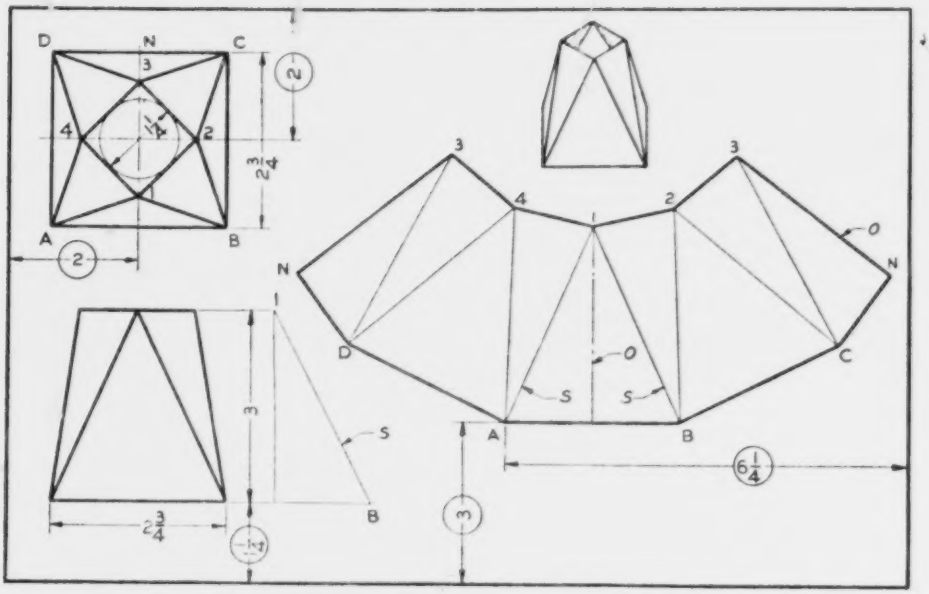
THREE-PIECE OFFSET



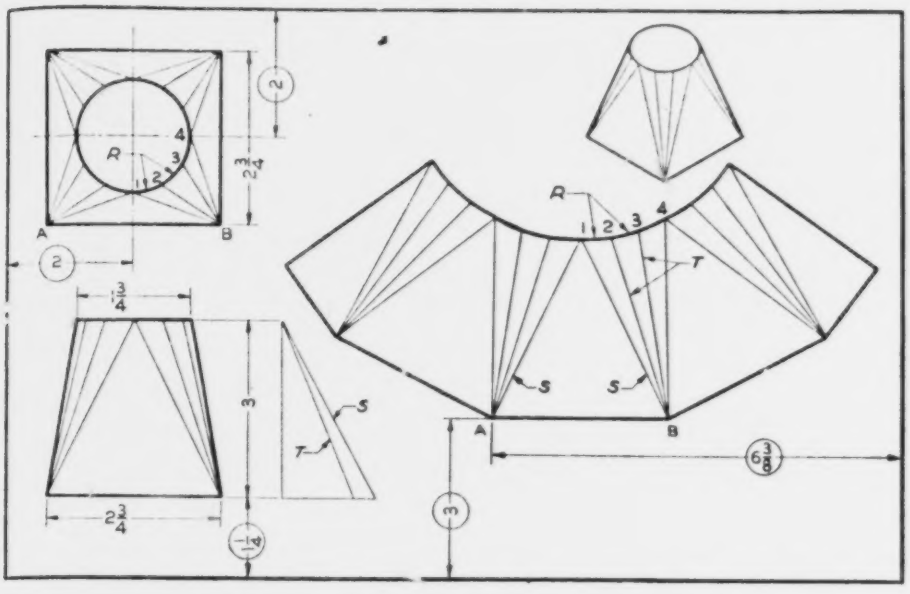
T JOINT



OFFSET TRANSITION PIECE



TRANSITION PIECE



SQUARE TO ROUND TRANSITION PIECE